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MẤT CÂN BẰNG ĐẾN ĐỘ BỀN MỖI ROTOR

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1. Lam Tran Thanh, Ngon Dang Thien, Cuong Le Chi, *Study on the Applicability of Influence Coefficient Method Combined with Vector Analysis in Dynamic Balancing Rigid Rotor Using Flexible Supports*, Proceedings 2018 4th International Conference on Green Technology and Sustainable Development, pp. 228-231, 2018.
2. Trần Thanh Lam, Đặng Thiện Ngôn, Lê Chí Cương, *Nghiên cứu, chế tạo, thực nghiệm mô hình rotor trục mềm*, Tạp chí KHGDKT Số 58, 2020.
3. Trần Thanh Lam, Đặng Thiện Ngôn, Trần Thái Sơn, Lê Chí Cương, *Nghiên cứu thực nghiệm đánh giá các thông số ảnh hưởng đến dao động của rotor*; Tạp chí KHGDKT, số 66/ 2021.
4. Thanh Lam Tran, Chi Cuong Le, Tien Hoang Ngo, and Ngoc Dang Khoa Tran, *An experimental study evaluating parameters effects on the vibration of rotor*, AIP Conference Proceedings 2420, 070003, 2021.
5. Thanh Lam Tran, Vinh Phoi Nguyen, Chi Cuong Le, Thien Ngon Dang, *Effect of unbalanced parameters on the fatigue behavior of transmission shaft in gearbox*, Strength, Fracture and Complexity; 18(2), pp.86-92, 2025 (ISI-Q4).
6. Thanh Lam Tran, Vinh Phoi Nguyen, Chi Cuong Le, Thien Ngon Dang, *Investigation on the Influence of Unbalanced Shaft Component in Gearbox on Displacement Using the Newmark- β Method*, PLOS ONE (SCIE – Q1), Revised following the second round of review and resubmitted.

Study on the Applicability of Influence Coefficient Method Combined with Vector Analysis in Dynamic Balancing Rigid Rotor Using Flexible Supports

Lam Tran Thanh, Ngon Dang Thien, Cuong Le Chi

Abstract - Dynamic balance using flexible supports is one of the commonly applied solutions to determine the position and unbalanced mass in rotors. However, dynamic balance using flexible supports are being determined experimentally, resulting in low accuracy and long calibration time. This paper presents some research results when applying the influence coefficient method to combine vector analysis (using balanced method on individual planes and trial mass). To verify experimentally the theory, HMC1501 sensors are attached at the supports to measure vibration induced displacement, an encoder 2.048 PPR is used to determine the position (phase angle ϕ and a DSPIC30F4011 microcontroller is equipped to receive signal were used. The measured results were obtained by the Fast Fourier Transform (FFT) combined with numerical analysis, including phase angle ϕ and unbalance mass U .

Keywords: dynamic balance machine; flexible supports; influence coefficient method; fast fourier transform; phase angle; unbalance mass.

I. INTRODUCTION

Dynamic balancing is a common method used to determine the unbalance of rotary components such as rotors, turbines, propellers, pumps and reduce the vibration during the operation of the equipment. Currently the dynamic balance systems for rotor includes rigid and flexible supports, in which the advantage of the flexible supports balance system is that it can balance the rotors at low speed [1, 2].

Dynamic balance system using flexible supports are based on experience and experiment having low accuracy as well as timely for calibration. In order to improve the accuracy of dynamic balance using flexible supports, some researches have been done [3 - 5].

This paper proposes a solution to improve accuracy of dynamic balance using flexible supports by applying the influence coefficients method [6, 7] combined with vector analysis. The position values (phase angle ϕ) and unbalance U ($U = m.r$) are determined on individual planes.

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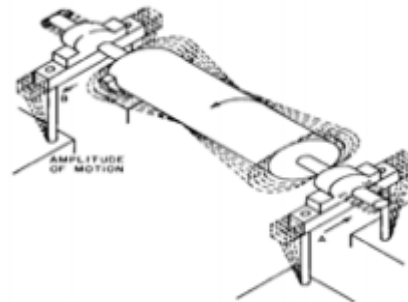


Figure 1. Principle of flexible supports balancing machine

II. EXPERIMENT MODELING

Figure 2 shows a rotor model, detecting the unbalance by converting the unbalance U to U_1, U_2 on two corresponding reference planes I and II near two supports. Thence, the sample rotor used for experiment is shown in Figure 3.

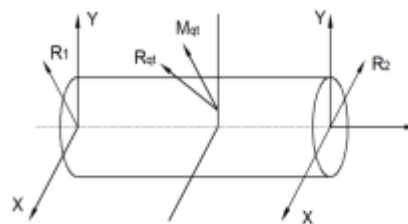


Figure 2. Balance to the two reference planes I and II

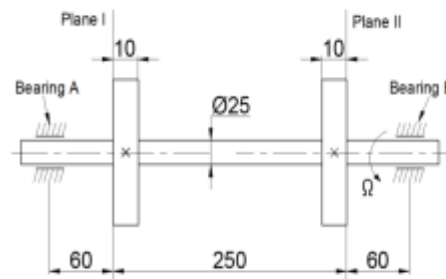


Figure 3. Dimension of sample rotor

Here, the plane I (near support A) and plane II (near support B) are chosen being the two reference planes. The unbalance U will be converted on these two planes. Let us consider the side view of the reference plan in Figure 4, where the unbalance is converted into a heavy spot with mass m_{eb} , radius R_U , and phase angle ϕ .

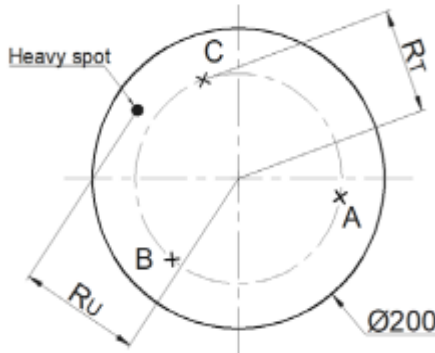


Figure 4. Reference Plan I

An experiment model for determining the unbalance on the disks was built as in Fig. 5, in which the disks have diameter $D_{max} = 200\text{mm}$ and thickness B of 10mm , at the heavy spot with a mass m_{cb} of 25g , $\phi = 45^\circ$ and radius R_u of 88mm . The balancing machine model consists of the following main components: a platform (for vibration and resonance resistance), a coupling, two flexible supports, two HMC-1501 sensors to measure vibration displacements, a position encoder (to measure the phase angle), a signal card using DSPIC30F4011 microcontroller, an electric control box (motor and speed control circuit) connected to a computer [8].

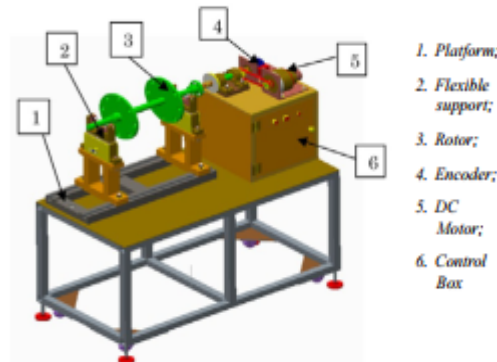


Figure 5. Dynamic balancing model using flexible supports

III. VIBRATION SIGNAL PROCESSING ANALYSIS

We have the displacement of the oscillation at the supports as a periodic function over time:

$$x(t) = X \sin(\omega t + \varphi)$$

where X is oscillation amplitude, ω is angular velocity of rotor (rad/s), and φ is initial phase angle.

Using the digital signal processing method [9], we obtain the raw vibration signal from support A is of the form, as shown in Figure 6.

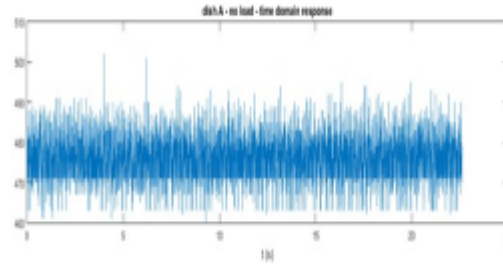


Figure 6. Vibration signal at support A by time domain

By using Fast Fourier Transform (FFT) to convert the vibration signal from the time domain to the frequency domain, we obtain the profile signal of support displacement, as shown in Figure 7.

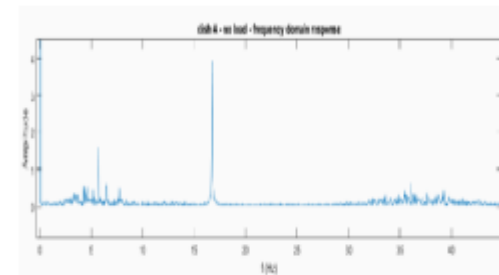


Figure 7. Vibration signal at support A by frequency domain

From Figure 7, it is found that the harmonic spectrum at frequency $f = 16.67\text{ Hz}$, corresponding to the velocity of $\omega = 104.87\text{ rad/s}$ or $n = 1000\text{ rpm}$. Therefore, in the signal filter, we will use a low pass filter with cut-off frequency $f_c = 20\text{ Hz}$. Then, the signal passing through the filter with four successive cycles is shown in Figure 8.

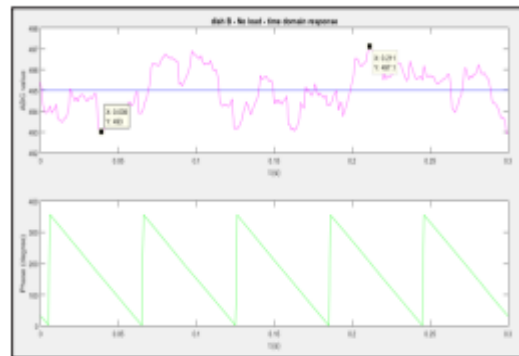
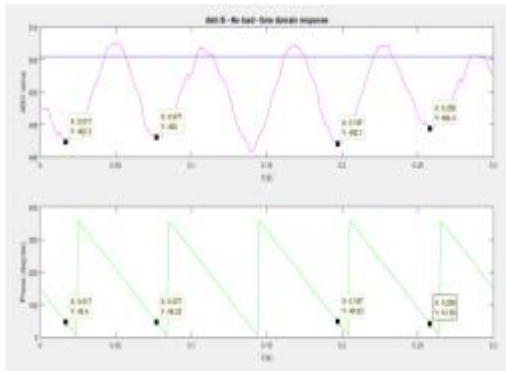


Figure 8. The initial vibration signal was processed at support A in time domain

IV. EXPERIMENT RESULT

A. Determination of the phase angle φ

By synchronizing the vibration signals (from sensors) and phase angle signals (from encoder) in real-time, we can determine the position (phase angle φ) as shown in Figure 9.


 Figure 9. Determine the unbalanced phase angle at plane I with $\varphi = 43^\circ$

B. Determination of the unbalance U

The system is supposed to be linear and X_1 is the amplitude of oscillation due to heavy spot. Adding trial mass m_T at a distance r_T , the amplitude X_m due to the trial mass and the heavy spot can be measured. Then, we determine the amount of unbalance U by the influence coefficient α at support A as follows [10]:

$$U = \frac{X_1}{X_m - X_1} m = \frac{X_m}{\alpha}$$

where α is the influence coefficient at support A, given by

$$\alpha = \frac{X_m - X_1}{m}$$

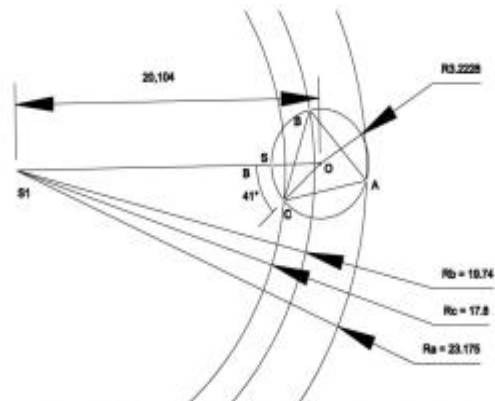
Using the vector analysis method with 3-test run to verify the parameters U and φ , we have the following results:

- At the three points A, B, and C, at 120° each other, as shown in Fig. 3, we attach $m_T = 20g$, $R_u = 88mm$ and calculate φ and U as shown in Table 1.

TABLE 1. THE RESULTS ARE MEASURED IN 3-POINT METHOD

Serial Number	Plan I		Plan II	
	Phase angle	Amplitude X_m	Phase angle	Amplitude X_m
Initial	0°	± 1	0°	± 1
Point A	"	23.18	"	9.14
Point B	"	19.74	"	7.13
Point C	"	17.80	"	8.43
Result	41°		22°	

Applied [11], we determine phase angle φ by 3-test run on individual planes (plan I) in Fig. 10, received result is $\varphi = 41^\circ$.


 Figure 10. Diagram for determining phase angle φ using 3-point method in plane I

- The weight of the unbalance is calculated by the following vector analysis formula [11]:

$$m_{cb} = m_T \cdot \frac{OS_1}{R_b} = 20 \cdot \frac{20.11}{19.7} = 20.5$$



Figure 11. Dynamic balancing machine

V. CONCLUSION

The following conclusions were made:

- Using Low-pass Filter with cut-off frequency $f_c = 20$ Hz has proven the choice of magnetic sensor HMC1501 for oscillation measurement in this method is fundamental.
- The experiments showed that the application of the influence coefficient method combined with vector analysis allows to quickly determine the position (phase angle φ) and the unbalance U. This method is also suitable for developing a software that determine the unbalance in real time instead of manual calculations methods.
- The experimental results also showed that the influence coefficients method combining the vector

analysis more accurately than the vector-based analysis method on three points A, B and C.

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NGHIÊN CỨU, CHẾ TẠO, THỰC NGHIỆM MÔ HÌNH ROTOR TRỤC MỀM RESEARCH, MANUFACTURE, EXPERIMENT ON FLEXIBLE ROTOR MODEL

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TÓM TẮT

Đảm bảo cho rotor vận hành ở tốc độ cao là một trong những yêu cầu cấp thiết hiện nay. Bài báo phân tích một số đặc tính cơ bản động lực học của rotor trên nền tảng mô hình Jeffcott rotor. Mô hình rotor trục mềm có thể vận hành ở tốc độ cao đến 12.000 vòng/phút được điều khiển bằng biến tần cho phép nghiên cứu các tốc độ tới hạn 1, tốc độ tới hạn 2 (mode 1 và mode 2) và cảm biến laser Keyence G35 được sử dụng để đo lường mất cân bằng (chuyển vị). Kết quả thực nghiệm trên mô hình đã xác định được tốc độ tới hạn (mode 1) lý thuyết là $n_{th} = 1115$ vòng/phút và kết quả thực nghiệm là $n_{th} = 1260$ vòng/phút với sai lệch < 15%. Điều này cho thấy mô hình thiết kế có thể ứng dụng trong thực tế phục vụ cho việc đánh giá hoạt động của rotor và cũng có thể được sử dụng để xác định tốc độ tới hạn 2 (mode 2).

Từ khóa: rotor trục mềm; tốc độ tới hạn; Jeffcott rotor; mất cân bằng; chuyển vị.

ABSTRACT

Ensuring the rotor to operate at high speed is one of the urgent requirements today. This paper analyzes some basic dynamics characteristics of the rotor on the basis of Jeffcott rotor model. The flexible rotor model that can operate at high speeds 12.000 rpm is controlled by inverter equipment that allows studying critical speeds 1, critical speed 2 (mode 1, mode 2) and laser Keyence G35 sensor used to displacement measure. Experimental results on the model have identified the theoretical critical speed (mode 1) is $n_{th} = 1115$ rpm and the experimental results are $n_{th} = 1260$ rpm with deviations < 15%. This shows that the design model can be applied in practice for the evaluation of rotor operation and can also be used to determine critical speed 2 (mode 2).

Keywords: Flexible rotor; critical speed; Jeffcott rotor; unbalance; displacement.

1. GIỚI THIỆU

Các thiết bị như máy nén, tuabin, máy bơm, động cơ phản lực, máy tăng áp... khi vận hành đều bị rung động do chuyển động của rotor. Nguyên nhân gây ra các rung động này thường là do rotor mất cân bằng trong khi chuyển động. Các thông số chính để đánh giá độ rung động của hệ thống là tốc độ của rotor (tốc độ tới hạn), độ ổn định của hệ thống và đáp ứng mất cân bằng [1].

Trên thế giới, nghiên cứu về động lực học rotor đã được nhiều nhóm nghiên cứu thực hiện, điển hình R. Tiwari [3] đã thực hiện các phân tích và nhận dạng các lỗi mất cân bằng trên hệ thống rotor. Tại Việt Nam,

nghiên cứu về phản ứng của tuabin trong nhà máy thủy điện khi vận hành được nhóm N. V. Khang [4] đề cập và nhóm P. H. Hoàng [5] thực hiện nghiên cứu ứng dụng mô hình rotor trục mềm để cân bằng trong phòng thí nghiệm.

Trong các nghiên cứu này, tốc độ tới hạn là tần số tự nhiên không bị suy giảm (không xét đến hệ số giảm chấn c) của hệ thống rotor được tập trung xem xét, được xem là thông số chính yếu để thiết kế các thiết bị quay. Do đó, cần thực hiện các phân tích, tính toán để xác định chính xác tốc độ tới hạn của hệ thống cùng hình dạng chế độ và phân bố năng lượng.

Thông thường tốc độ tối hạn được mong muốn ở mức 10% đến 20% trên hoặc dưới phạm vi tốc độ hoạt động của rotor [1]. Tuy nhiên, có nhiều rotor sử dụng các ổ đỡ có giảm chấn đặc biệt có thể hoạt động trên tốc độ tối hạn 2 (mode 2) bằng cách thay đổi các hệ số ảnh hưởng của gối đỡ, điều này dẫn đến các vấn đề mất ổn định nghiêm trọng khi hệ thống chuyển sang hoạt động ở tốc độ tối hạn 1 (mode 1).

Bài báo trình bày các kết quả nghiên cứu điều khiển tốc độ vô cấp của rotor (0 – 12.000 vòng/phút) và đáp ứng tín hiệu chuyển vị tương ứng sử dụng mô hình rotor trục mềm.

2. NỘI DUNG NGHIÊN CỨU

2.1 Thông số thiết kế

Từ yêu cầu thực tế, để nghiên cứu đánh giá các thông số gây mất cân bằng trên rotor trục mềm, ta cần xây dựng 1 mô hình thực nghiệm để kiểm chứng. Mô hình được lựa chọn là Jeffcott rotor, đây là mô hình rotor được sử dụng nhiều nhất để mô tả động lực học và cho phép phân tích phản ứng khi rotor hoạt động ở tốc độ cao.

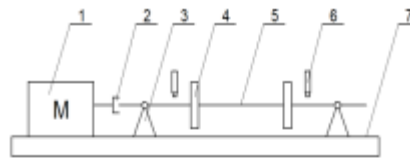
Do vậy, thông số đầu vào của mô hình thiết kế bao gồm:

- Phạm vi điều khiển tốc độ: 0 – 12.000 vòng/phút.
- Khả năng chuyển đổi và xử lý tín hiệu analog sang tín hiệu số.
- Khả năng nhận biết và ghi nhận lượng mất cân bằng thông qua tín hiệu chuyển vị.
- Máy cứng vững, rung động ít.

2.2 Nguyên lý thiết kế

Với thông số như trên, khi rotor quay, biên độ dao động sẽ xuất hiện dọc trên chiều dài làm việc của rotor. Các giá trị biên độ dao động này sẽ được các cảm biến thu nhận, chuyển đổi thành các tín hiệu số đưa về bộ xử lý trung tâm để tính toán, xử lý.

Phác thảo sơ đồ nguyên lý của mô hình thiết kế như ở hình 1.

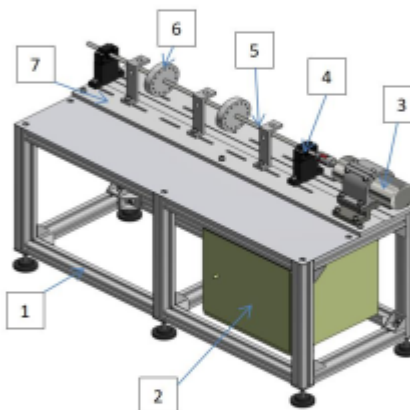


1. Động cơ – 2. Khớp nối mềm – 3. Gối đỡ
 4. Đĩa – 5. Trục – 6. Cảm biến – 7. Đế

Hình 1. Phác thảo mô hình rotor trục mềm

2.3 Kết cấu

Từ sơ đồ nguyên lý, phương án thiết kế kết cấu mô hình rotor trục mềm được đề xuất như ở hình 2.



1. Khung máy – 2. Tủ điều khiển – 3. Động cơ
 4. Cụm gối đỡ – 5. Trục – 6. Đĩa – 7. Đế

Hình 2. Thiết kế kết cấu mô hình rotor trục mềm

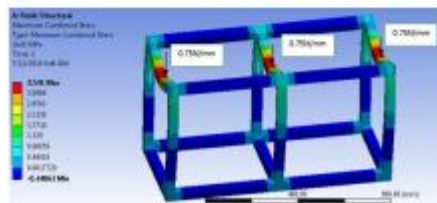
Mô hình có đơn giản hóa một số thành phần nhưng vẫn thể hiện đầy đủ các yếu tố cấu thành chính của một hệ rotor điển hình trong thực tế. Đặc biệt là vẫn thể hiện tốt các đặc điểm quan trọng của động lực học rotor là: tốc độ và mất cân bằng. Mô hình thiết kế gồm các thành phần chính: 1 trục trơn, 2 đĩa để bố trí khối lượng, 2 cụm gối đỡ, cơ cấu truyền động, thiết bị dẫn động, thiết bị đo tín hiệu chuyển vị và cụm đế máy, khung máy.

Mô hình là trục đối xứng và sử dụng ổ bi cơ nên bỏ qua hiệu ứng giảm chấn liên quan



2.4 Kiểm nghiệm thiết kế

Để và kết cấu khung máy bằng thép là kết cấu chính chịu tải trọng của mô hình nên cần được tính toán kiểm nghiệm bền để có khả năng chống rung và ngăn ngừa hiện tượng cộng hưởng. Kết quả tính toán, phân tích chuyên vị của khung máy ở trạng thái động lực học được trình bày ở hình 4 [6].



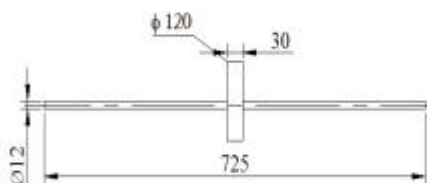
Ta thấy $[\sigma]_{\max} = 3,54 \text{ N/mm}^2 > \sigma_{\text{load}} = 3 \times 0,75 = 2,25 \text{ N/mm}^2$: do vậy kết cấu chắc chắn, không rung.

Trên cơ sở thiết kế và phương án kết cấu đã được kiểm nghiệm, một mô hình rotor trục mềm được chế tạo như ở hình 5 với các thông số kỹ thuật chính như sau:

- Đường kính đĩa $D_{\text{đĩa}} = 120 \text{ mm}$;
- Chiều dài trục quay $L_{\text{max}} = 725 \text{ mm}$;
- Khối lượng trục $m_{\text{trục}} = 640 \text{ g}$;
- Khối lượng đĩa $m_{\text{đĩa}} = 2150 \text{ g}$;
- Cảm biến đo chuyển vị LK_G35;
- Động cơ Spinder 3 pha 220 VAC, 1.5 kW;



Rotor trong mô hình chạy thực nghiệm là 1 đĩa đơn được gắn đồng tâm trên một trục đàn hồi đồng nhất.



**Hình 7. Mô hình vận hành tại tốc độ tới hạn
I (mode I)**

Đáp ứng không cân bằng của rotor được mô tả bởi phương trình chuyển động bậc hai của hệ thống như sau [2]:

$$m\ddot{X} + c\dot{X} + kX = M\omega^2 \cos \omega t$$

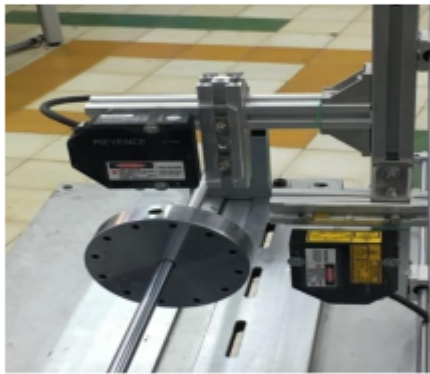
Trong đó, bỏ qua hệ số giảm chấn c , công thức xác định tốc độ tới hạn 1 (mode 1):

$$\omega_1 = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2.146,5 \cdot 10^5}{2150}} = 116,7 \frac{\text{rad}}{\text{s}}$$

Khi đó, số vòng quay của rotor sẽ là:

$$n_{1/t} = \frac{60 \cdot \omega_1}{2\pi} = \frac{60 \cdot 116,7}{2 \cdot 3,14} = 1115 \text{ v/ph}$$

Để đo lường mất cân bằng, cảm biến LK_G35 được bố trí như ở hình 8.



Hình 8. Bố trí lắp đặt Sensor LK_G35

Công việc thực nghiệm được tiến hành với nhiều cấp tốc độ khác nhau: $n = 600, 900, 960, \dots, 6000$ (vòng/phút), qua đó đo và đánh giá tín hiệu dao động thu được (đặc biệt là tín hiệu nhiễu).

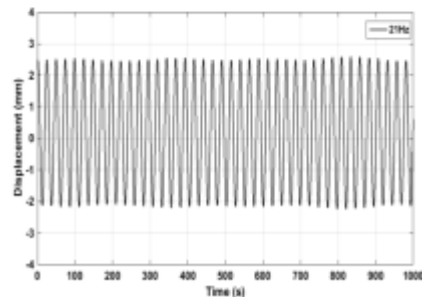
Kết quả đo đạc cho thấy tín hiệu đo dao động thu được ổn định và các giá trị tương ứng được trình bày trong bảng 1.

Bảng 1. Kết quả thực nghiệm đo dao động

STT	Tần số (Hz)	Tốc độ (vòng/phút)	Chuyển vị (mm)
1	10	600	$\pm 0,3$
2	15	900	$\pm 0,3$
3	16	960	$\pm 0,39$

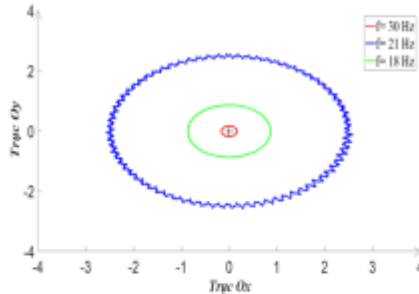
STT	Tần số (Hz)	Tốc độ (vòng/phút)	Chuyển vị (mm)
4	17	1020	$\pm 0,57$
5	18	1080	$\pm 0,87$
6	19	1140	$\pm 1,2$
7	20	1200	$\pm 1,38$
8	21	1260	$\pm 2,32$
9	22	1320	$\pm 0,55$
10	23	1380	$\pm 0,39$
11	24	1440	$\pm 0,31$
12	25	1500	$\pm 0,26$
13	26	1560	$\pm 0,22$
14	27	1620	$\pm 0,20$
15	28	1680	$\pm 0,14$
16	30	1800	$\pm 0,15$
17	40	2400	$\pm 0,10$
18	50	3000	$\pm 0,08$
19	60	3600	$\pm 0,06$
20	70	4200	$\pm 0,05$
21	100	6000	$\pm 0,04$

Kết quả đo cho thấy tốc độ tới hạn 1 (mode 1) đạt được khi $n = 1260$ vòng/phút (tương ứng tần số $f = 21$ Hz) với chuyển vị theo phương X là $X_{\max} = \pm 2,32$ mm.



Hình 9. Tín hiệu thu được tại tốc độ tới hạn thực tế $n = 1260$ vòng/phút ($f = 21$ Hz)

Quỹ đạo chuyển động của rotor được thể hiện trên hình 10 tại 3 tần số lần lượt là $f = 18 \text{ Hz}$, $f = 21 \text{ Hz}$ (mode 1) và $f = 30 \text{ Hz}$.



Hình 10. Quỹ đạo chuyển động của tâm trục tương ứng tần số $f = 18\text{Hz}$, 21Hz và 30Hz

Từ các kết quả tính toán (1) ta có tốc độ tới hạn (mode 1) là $n_{01} = 1115$ vòng/phút và kết quả thực nghiệm (bảng 1) là $n_{01} = 1260$

vòng/phút cho thấy các ứng xử của mô hình phù hợp (sai lệch $< 15\%$) [1].

3 KẾT LUẬN

Mô hình thiết kế và chế tạo đã được kiểm nghiệm thành công, đảm bảo hệ thống vận hành ổn định. Kết quả đo cho thấy tốc độ tới hạn 1 (mode 1) đạt được khi $n = 1260$ vòng/phút (tương ứng tần số $f = 21 \text{ Hz}$) với chuyển vị theo phương X là $X_{\max} = \pm 2,32 \text{ mm}$.

Mô hình sử dụng cảm biến LK_G30 + bộ Controller G3000 + Software Keyence thu thập tín hiệu đo dao động có độ chính xác và tốc độ xử lý cao.

Mô hình là tiền đề để phát triển việc tính toán, đo kiểm cho rotor vận hành tại tốc độ tới hạn 2 (mode 2) và các phản ứng của rotor thu được tại tốc độ tới hạn 2.

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NGHIÊN CỨU THỰC NGHIỆM ĐÁNH GIÁ CÁC THÔNG SỐ ẢNH HƯỞNG ĐẾN DAO ĐỘNG CỦA ROTOR EXPERIMENT STUDY EVALUATING PARAMETERS EFFECTS ON THE VIBRATION OF ROTOR

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TÓM TẮT

Rung động trong trục quay là một hiện tượng phổ biến thường gặp trong các hệ thống rotor. Việc xác định các yếu tố gây rung động để qua đó thực hiện các biện pháp giảm rung nhằm đảm bảo cho rotor vận hành ổn định là một trong những yêu cầu cấp thiết hiện nay. Trong bài báo này, một mô hình đánh giá rung động của rotor đã được thiết lập cho phép khảo sát các thông số chính ảnh hưởng đến rung động của rotor bao gồm tốc độ tới hạn, độ lệch tâm gây ra do mất cân bằng và góc pha ban đầu. Các kết quả thực nghiệm cho thấy: khi tốc độ quay gần tốc độ tới hạn sự rung động trở nên không ổn định và biên độ dao động tăng mạnh; lượng mất cân bằng tăng dẫn đến biên độ dao động tăng; góc pha ban đầu của vị trí đặt tải nếu bị thay đổi trong quá trình hoạt động sẽ dẫn đến các biến đổi khác nhau của giai đoạn rung động dẫn đến sự tăng hoặc giảm các ứng suất uốn có thể dẫn đến hiện tượng mỏi xuất hiện trên trục quay.

Từ khóa: rotor; dao động; tốc độ tới hạn; mất cân bằng; góc pha ban đầu.

ABSTRACT

Vibration in the rotary shaft is a common phenomenon observed in rotor systems. Determining the factors that cause the vibration to implement the vibration reduction measures to ensure the rotor running stably is one of the urgent requirements today. In this paper, a model of rotor vibration assessment has been proposed which allows to investigate the main parameters influencing rotor vibrations including critical speed, unbalance eccentricity and initial phase angle. The experimental results show that: when the speed is near the critical speed, the vibration becomes unstable and the amplitude of the oscillation increases sharply; the amount of unbalance increases leading to an increase in amplitude of oscillation; the initial phase angle of the load position, if changed during operation, will make different oscillation periods leading to an increase or decrease in bending stress leading to rotor fatigue.

Keywords: rotor; vibration; critical speed; unbalance; initial phase angle.

1. GIỚI THIỆU

Máy quay là một loại cơ cấu cơ khí quay phổ biến, được sử dụng rộng rãi trong các thiết bị công nghiệp. Hoạt động của cơ cấu quay này ảnh hưởng lớn đến khả năng vận hành ổn định của máy, nếu gây ra rung động quá mức cho phép có thể ảnh hưởng đến chất lượng sản phẩm và cuối cùng có thể gây ra hỏng hóc [1].

Việc hỏng hóc của cơ cấu quay thường gặp trong các thiết bị như cụm truyền chuyển động của các phương tiện giao thông, tuabin, máy bơm,... đòi hỏi kinh phí sửa chữa lớn và quan trọng hơn là có thể gây nguy hiểm đến tính mạng con người. Nguyên nhân gây ra rung động là do một số yếu tố tác động lên rotor trong khi vận hành. Do vậy, việc nghiên cứu động lực học của rotor để xác định các thông số chính ảnh hưởng đến rung động của

Doi: <https://doi.org/10.54644/jte.66.2021.1060>

Bảng 1. Các thành phần nguyên tố của thép C45

Nguyên tố	C	Si	Mn	P	S	Ni	Cr	Cu
Thành phần (%)	0,46	0,21	0,64	0,021	0,007	0,04	0,09	0,16

Đề xuất cân bằng rotor theo phương pháp sử dụng 2 mặt phẳng qui đổi, mẫu quay có kết cấu như hình 3. [3]



Hình 3. Trục và 2 đĩa gắn khối lượng thử

Các thông số kỹ thuật chính mẫu quay như sau:

- Đường kính đĩa, $D_{disc} = 100 \text{ mm}$
- Chiều dài trục quay, $L_{max} = 256 \text{ mm}$
- Khối lượng trục, $m_{shaft} = 300 \text{ g}$
- Khối lượng đĩa, $m_{disc} = 180 \text{ g}$
- Bề dày đĩa, $B = 20 \text{ mm}$

2.2 Mô hình toán học

Phương trình chuyển động cho rotor (hệ thống trục và đĩa) như sau:

$$M\ddot{u} + C\dot{u} + Ku = F(t) \quad (1)$$

Phương trình chuyển động theo các phương x, y sẽ là:

$$M\ddot{x} + C\dot{x} + Kx = M\omega^2 \cos \omega t$$

$$M\ddot{y} + C\dot{y} + Ky = M\omega^2 \sin \omega t$$

Hay có thể viết lại:

$$M\ddot{x} + C\dot{x} + Kx = m\omega^2 \cos \omega t \quad (2)$$

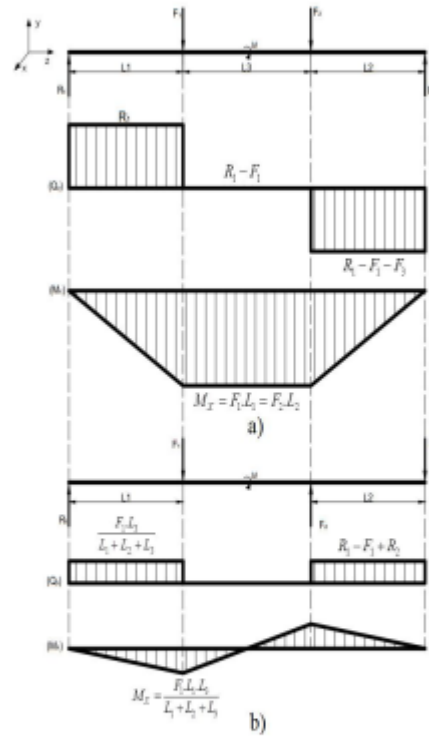
$$M\ddot{y} + C\dot{y} + Ky = m\omega^2 \sin \omega t \quad (3)$$

Trong đó:

- M : khối lượng của rotor
- C : hệ số giảm chấn của hệ thống
- K : độ cứng của rotor

- m : khối lượng thử thêm vào
- r : bán kính đặt khối lượng thử
- ω : tốc độ quay của rotor

Sơ đồ lực tác động lên trục biểu diễn ở hình 4. Với các vị trí đặt tải khác nhau: $0^\circ - 0^\circ$, $0^\circ - 90^\circ$, $0^\circ - 180^\circ$ ta có được biểu đồ phân bố lực tương ứng.

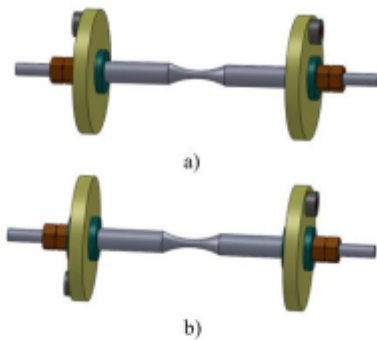


Hình 4. Biểu đồ phân bố lực và moment tác dụng lên rotor: (a) vị trí $0^\circ - 0^\circ$ và (b) vị trí $0^\circ - 180^\circ$

Ta có ứng suất uốn sinh ra do lực li tâm F_c là:

$$\sigma_u = \frac{M_u}{W_u} = \frac{f(F_c)}{\pi d^3} \quad (4)$$

Do đó, nếu giá trị lực li tâm $F_c = m\omega^2$ tác động lên trục không đổi thì vị trí tải tác động lớn nhất tương ứng với cách đặt tải đối xứng ($0^\circ - 0^\circ$) và vị trí tải tác động nhỏ nhất tương ứng vị trí $0^\circ - 180^\circ$.



Hình 5. Trục mẫu tại vị trí đặt tải:
 (a) $0^\circ - 0^\circ$ và (b) $0^\circ - 180^\circ$

2.3 Phương pháp đo lường

Mô hình sử dụng động cơ tích hợp trục chính có số vòng quay $n = 24.000$ vòng/phút và được điều khiển bởi biến tần. Tốc độ thực tế của rotor được xác định thông qua bộ đọc Encoder. Dao động của trục quay được đo bằng cảm biến laser Keyence LK-G35 có sai số $\pm 0,001\text{mm}$; khoảng cách đo là 30mm. Để xác định lượng mất cân bằng trên rotor ta sử dụng cảm biến gia tốc và phần mềm tính toán Digivibe MX của Hãng Erbsed Instruments. Giá trị hiển thị trên phần mềm là giá trị trung bình bình phương RMS của đại lượng $G = \epsilon_p \cdot \omega$ (mm/s) như hình 6.



Hình 6. Thiết bị đo mất cân bằng của hãng Erbsed Instruments

3. KẾT QUẢ

3.1 Ảnh hưởng do tốc độ quay (tốc độ tới hạn)

Rotor khi vận hành ở gần tốc độ tới hạn thứ 1 (mode 1) sẽ có dạng dao động như hình 7.



Hình 7. Mô hình rotor vận hành tại tốc độ tới hạn thứ 1 (mode 1)

Từ (1), bỏ qua hệ số giảm chấn C , ta có công thức xác định tốc độ tới hạn thứ 1 [11]:

$$\omega_c = \sqrt{\frac{k}{m}} = \sqrt{\frac{656,3 \cdot 10^5}{660}} = 315 \frac{\text{rad}}{\text{s}}$$

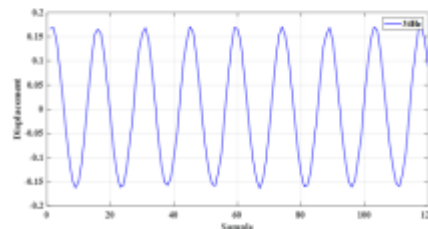
Khi đó, số vòng quay của rotor sẽ là:

$$n_c = \frac{60 \cdot \omega_1}{2\pi} = \frac{60 \cdot 315}{2,3,14} = 3009 \text{ vg/ph}$$

Để đảm bảo rotor trong mô hình vận hành ổn định và tương ứng với các tốc độ quay thực tế của các thiết bị; ta chọn dải tốc độ thí nghiệm của rotor là $n_1 = 800$ vòng/phút, $n_2 = 1500$ vòng/phút và $n_3 = 2000$ vòng/phút (với $n_3 = 2000$ vòng/phút $< 70\% \cdot n_c = 2106$ vg/ph). Kết quả đo đạc thực nghiệm được trình bày ở Bảng 2.

Bảng 2. Kết quả thực nghiệm đo dao động

STT	Tần số (Hz)	Tốc độ trung bình (vòng/phút)	Chuyển vị	
			X (mm)	Y (mm)
1	19	800	$\pm 0,05$	$\pm 0,12$
2	26	1500	$\pm 0,055$	$\pm 0,11$
3	34	2000	$\pm 0,05$	$\pm 0,10$
4	51	3000	$\pm 0,08$	$\pm 0,18$



Hình 8. Kết quả đo chuyển vị tại tốc độ thực tế $n_3 = 2000$ vòng/phút ($f = 34 \text{ Hz}$)

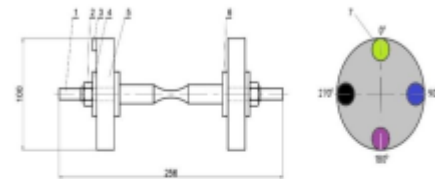
Từ kết quả đo cho thấy: khi rotor vận hành ở tốc độ dưới tốc độ tới hạn thứ 1; dao

động trên rotor là ổn định (dao động trục $X = \pm 0.05$; trục $Y = \pm 0.11$); vận hành tại tốc độ tối hạn thứ 1: dao động tăng mạnh ($X = \pm 0.08$; $Y = \pm 0.18$).

3.2 Ảnh hưởng do mất cân bằng

3.2.1 Trường hợp 1 - Khối lượng thử đặt đối xứng (vị trí $0^\circ - 0^\circ$)

Rotor khi mất cân bằng có kết cấu như hình 9.



1. Trục quay; 2. Đai ốc; 3. Khối lượng thử; 4, 6. Bạc lót; 5. Đĩa; 7. Vị trí đặt tải

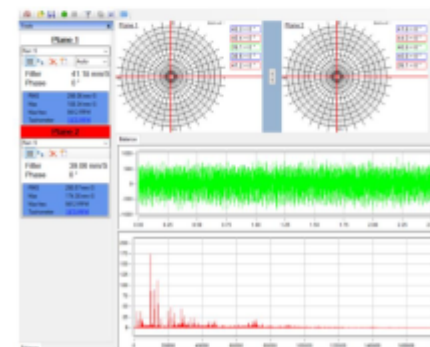
Hình 9. Rotor thực nghiệm mất cân bằng

Theo tiêu chuẩn mất cân bằng ISO 1940/1 – 2013; ta tính toán khối lượng thử tương ứng các cấp độ mất cân bằng lần lượt là G2.5, G6.3 và G16 [1].

Đối với G16, ta có khối lượng thử cho phép tính toán tại tốc độ tối thiểu n_1 như sau:

$$m_{trial} = \frac{k \cdot 9,54 \cdot G_{16} \cdot M_{rotor}}{n_1 \cdot r} = 37,7 \text{ g}$$

Vì thế, ta chọn các khối lượng thử lần lượt 10g, 20g và 30g và chạy theo các tốc độ đã đề xuất, kết quả đo như hình 10 [12].

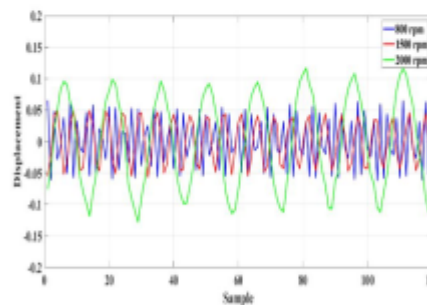


Hình 10. Kết quả đo cân bằng trên 2 mặt phẳng tại vị trí $0^\circ - 0^\circ$; $m_{trial} = 30g$.

Kết quả đo lường mất cân bằng trên rotor tại vị trí $0^\circ - 0^\circ$ với các giá trị tải và tốc độ tương ứng được trình bày trong bảng 3.

Bảng 3. Kết quả đo mất cân bằng khi gắn khối lượng thử tại vị trí $0^\circ - 0^\circ$

STT	Khối lượng thử (gram)	Tốc độ (vòng/phút)	$G = e_p \cdot \omega$ (mm/s)
1	0	800	0.58
2	0	1500	1.12
3	0	2000	1.34
4	10	800	0.9
5	10	1500	4.9
6	10	2000	20.5
7	20	800	1.1
8	20	1500	9.4
9	20	2000	32.8
10	30	800	2.2
11	30	1500	14.3
12	30	2000	42.3



Hình 11. Kết quả đo chuyển vị tại $m_{trial} = 20g$; tốc độ 800, 1500 và 2000 vòng/phút

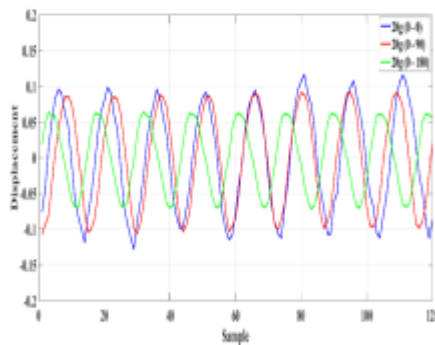
Nhận xét: tại tốc độ 2000 vòng/phút dao động của rotor khi mất cân bằng là lớn nhất.

3.2.2 Trường hợp 2 - Khối lượng thử đặt không đối xứng (vị trí $0^\circ - 90^\circ$; $0^\circ - 180^\circ$)

Thay đổi vị trí đặt tải ban đầu (góc pha), ta thu được các kết quả mất cân bằng như bảng 4.

Bảng 4. Kết quả đo mất cân bằng khi gắn khối lượng thử tại vị trí $0^\circ - 90^\circ, 0^\circ - 180^\circ$

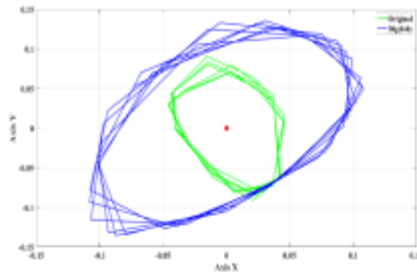
STT	Vị trí đặt m_{trial}	Khối lượng thử (gram)	Tốc độ (vòng/phút)	$G = e_p \cdot \omega$ (mm/s)
1	$0^\circ - 0^\circ$	10	2000	20.5
2	$0^\circ - 90^\circ$	10	2000	14.3
3	$0^\circ - 180^\circ$	10	2000	2.6
4	$0^\circ - 0^\circ$	20	2000	32.8
5	$0^\circ - 90^\circ$	20	2000	28.7
6	$0^\circ - 180^\circ$	20	2000	3.7
7	$0^\circ - 0^\circ$	30	2000	42.3
8	$0^\circ - 90^\circ$	30	2000	35.4
9	$0^\circ - 180^\circ$	30	2000	4.2

**Hình 12.** Kết quả đo chuyển vị tại $m_{trial} = 20g, n = 2000$ vòng/ph, các vị trí $0^\circ - 0^\circ, 0^\circ - 90^\circ, 0^\circ - 180^\circ$.

Kết quả đo cho thấy lượng mất cân bằng và chuyển vị giảm dần từ vị trí góc pha $0^\circ - 0^\circ \rightarrow$ vị trí $0^\circ - 90^\circ \rightarrow$ vị trí $0^\circ - 180^\circ$.

3.3 Quỹ đạo chuyển động

Quỹ đạo chuyển động của rotor được thể hiện trên hình 13 tại tốc độ $n_3 = 2000$ vòng/phút; trong 2 trường hợp rotor không tải và rotor có tải 30g, vị trí $0^\circ - 0^\circ$.

**Hình 13.** Quỹ đạo chuyển động của tâm trục tương ứng rotor không tải và có tải 30g

Kết quả thực nghiệm cho thấy quỹ đạo chuyển động của rotor ứng xử phù hợp qui luật với các thông số ảnh hưởng.

4. KẾT LUẬN

Mô hình đánh giá rung động rotor đã được thiết lập và vận hành ổn định. Ảnh hưởng của nhiều tham số đến đáp ứng rung động, bao gồm tốc độ quay, độ lệch tâm không cân bằng và góc pha ban đầu được nghiên cứu chi tiết.

Kết quả cho thấy rằng:

Khi tốc độ quay của rotor gần tốc độ tới hạn, sự rung động trở nên không ổn định, biên độ dao động tăng mạnh.

Cùng đó, nếu rotor bị mất cân bằng sẽ dẫn đến biên độ dao động tăng nhanh làm rotor nhanh chóng hư hỏng.

Góc pha ban đầu của vị trí đặt tải nếu bị thay đổi trong quá trình hoạt động sẽ dẫn đến các biến đổi khác nhau của giai đoạn rung động dẫn đến sự tăng hoặc giảm các ứng suất uốn có thể dẫn đến hiện tượng mỏi xuất hiện trên trục quay.

Các phát hiện này có ý nghĩa quan trọng trong việc định hướng thiết kế an toàn giúp rotor vận hành trong thời gian dài và hỗ trợ nghiên cứu sâu hơn về dự đoán độ bền mỏi cho rotor.

LỜI CẢM ƠN

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An Experimental Study Evaluating Parameters Effects on the Vibration of Rotor

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Abstract. Vibration in the rotary shaft is a common phenomenon observed in rotor systems. Determining the factors that cause the vibration to implement the vibration reduction measures to ensure the rotor is running stably is one of the urgent requirements today. In this paper, a machine model of rotor vibration assessment has been proposed, examining the main parameters influencing rotor vibrations, including critical speed, unbalance eccentricity and orientation angle. The experimental results show that: when the speed is around the critical speed, the vibration becomes unstable and the amplitude of the oscillation increases sharply; the orientation phase angle position causes different variations of the vibrating period leading to the imposed or superposition of tensile or compressive stresses resulting in fatigue occurring on the rotor. The present study illustrates the applicability of employing simple models to predict the dynamic response of a simple rotor system with acceptable accuracy.

INTRODUCTION

Rotary machinery is a common type of rotating mechanical mechanism widely used in industrial equipment. The operation of this rotating mechanism greatly affects the stable operation of the machine. If it causes vibration, it can affect the quality of the product and may eventually cause damage.

The failure of the rotating mechanism is common in applications such as aviation engines, turbines, pumps, etc., requiring large repair costs and, more importantly, can endanger human life.

The cause identified here is because the rotor is unstable, so when operating, it causes vibration.

Therefore, it is important to study rotor dynamics to determine the amount of rotor unbalance through main parameters such as rotor speed (critical speed), system stability and unbalance response has been and is being studied [1]

In there, critical speed and unbalance are the two main parameters most studied and evaluated [1-8]. The critical rotor speeds are the speeds at which the rotor deforms the most (resonance). Normally the critical speed is desired at 20% to 30% over the rotor operating speed range [2]. However, many rotors use specially damped bearings that can operate above critical speed 2 (mode 2) by changing the bearing influence factors, which leads to problems serious instability when the rotor switches to critical speed 1 operation (mode 1)

Rotor unbalances the uneven distribution of mass around the rotor's center of rotation. There are many causes of rotor unbalance, such as manufacturing, assembly, deformation or wear during operation.

R. Tiwari performed the analysis and identification of unbalance errors on the rotor system [3]. B. Xu proposes a balanced method without a trial run [4]. Y. A. Khulief offers a method that does not require additional trial mass on the rotor when balancing [5]. Yuanping Xu considered using active magnetic bearing control in balance [6]. Shachar

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Tresser offers another approach in the field of balance that is low speed running combined with external excitation Force [7]. Recently Guangfu Bin suggested balancing using 3 or more planes to improve accuracy [8].

From the above survey results, this paper present the main contents: setting up a typical rotor model, consisting of a rotary shaft and discs, allowing experimental measurements to evaluate the vibrational states of the rotors based on parameters such as rotational speed, unbalanced eccentricity and initial phase angle.

EXPERIMENT MODELING

The rotating detail is proposed to be designed and manufactured according to ISO 1143:2010 [10].

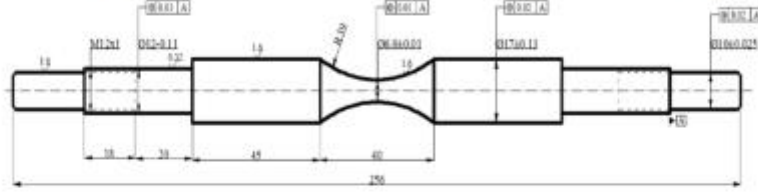


FIGURE 1. C45 steel rotating template shaft

It is proposed to evaluate the unbalanced rotor by the 2 plane method [2], so the rotating template has the structure as shown in Figure 2.

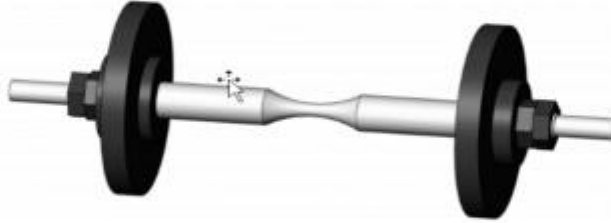


FIGURE 2. Shaft and 2 disc for set trial mass

The main specifications of the template rotor are as follows:

- Diameter of disc $D_{disc} = 100$ mm;
- Spindle length $L_{max} = 256$ mm;
- Shaft mass $m_{shaft} = 300$ g;
- Disk mass $m_{disc} = 180$ g;
- Disc thickness $B = 20$ mm;

DESIGN AND SIMULATION

Figure 3 shows the model focuses on evaluating the common oscillation of the rotating shaft as transverse vibration (ignoring torsional and axial vibration).

Mathematical Model

The equation of motion for the rotor (shaft and disc system) is:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\} \quad (1)$$

The equation of motion in the x, y directions will be:

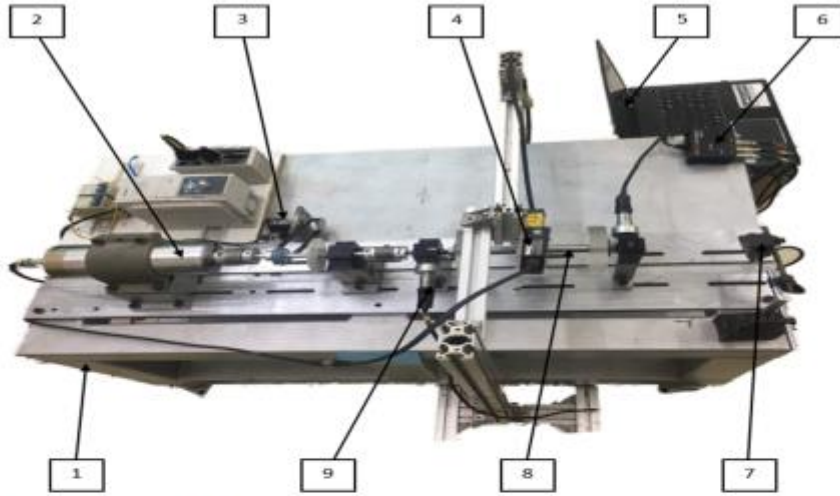
$$M\ddot{x} + C\dot{x} + Kx = M\omega^2 \cos\omega t \quad (2)$$

$$M\ddot{y} + C\dot{y} + Ky = M\omega^2 \sin\omega t \quad (3)$$

It can rewrite:

$$M\ddot{x} + C\dot{x} + Kx = mr\omega^2 \cos\omega t \quad (4)$$

$$M\ddot{y} + C\dot{y} + Ky = mr\omega^2 \sin\omega t \quad (5)$$



1. Frame – 2. Motor – 3. Encoder – 4. Displacement Sensor – 5. Computer – 6. DAQ – 7. Optical Sensor – 8. Rotor – 9. Accelerometer

FIGURE 3. Experimental model for vibration measurement

Where M , C , K are the mass, damping coefficient, and stiffness of the rotor, respectively; m , r , ω are the trial mass, the radius of the trial mass and the rotational speed, respectively.

The diagram of the force acting on the shaft is shown in Figure 4. With different load positions: 0° - 0° , 0° - 90° , 0° - 180° , we get the corresponding force distribution chart.

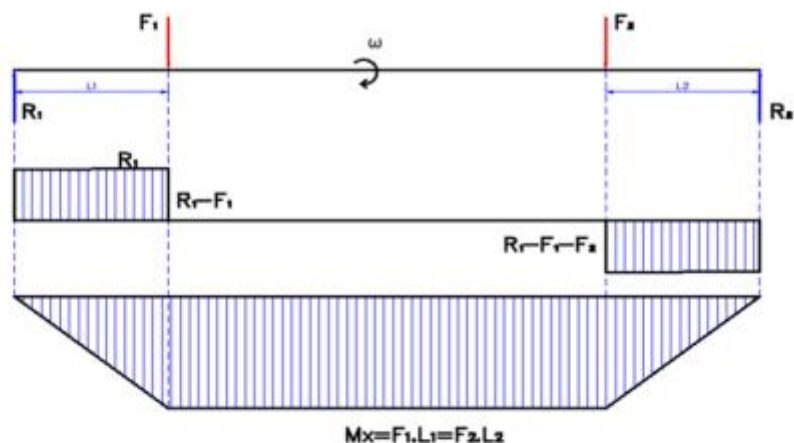


FIGURE 4. Schematic diagram of the force distribution on the rotor at position $0^\circ - 0^\circ$

Therefore, if the force value is constant, the position of the maximum impact force corresponds to the symmetrical loading ($0^\circ - 0^\circ$) and the position of the smallest impact force is the position $0^\circ - 180^\circ$.

Measurement Method

The model uses a spindle integrated motor with a rotational number of $n = 24,000$ rpm and is controlled by an inverter. The actual speed of the rotor is determined through the Encoder. Vibrations of the rotating shaft are detected and measured by the Keyence LK-G35 laser sensor. The determining the amount of unbalance on the rotor, we use the acceleration sensor and the Digivibe MX calculation software of Erbesed Instruments, as shown in Figure 5.



FIGURE 5. Measuring device of Erbesed Instrument

RESULT

Effect of rotary speed (Critical speed)

The rotor when operating near the critical speed 1 (mode 1) will have the profile as shown in figure 6.

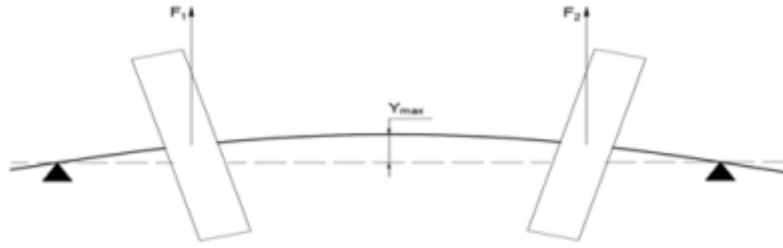


FIGURE 6. Model of rotor operating at critical speed 1 (mode 1)

In which, ignoring the damping coefficient c , the formula for determining the critical speed 1 [9]:

$$\omega_1 = \sqrt{\frac{k}{m}} = \sqrt{\frac{656,3 \cdot 10^3}{660}} = 315 \left(\frac{\text{rad}}{\text{s}} \right) \quad (6)$$

Then the speed of rotor will be:

$$n_1 = \frac{60 \cdot \omega_1}{2\pi} = \frac{60 \cdot 315}{2 \cdot 3,14} = 3009 \text{ (rpm)} \quad (7)$$

To ensure that the rotor in the model operates stably and corresponds to the actual rotational speeds of the equipment;

We choose the experimental speed range of the rotor as $n_1 = 800$ rpm, $n_2 = 1500$ rpm and $n_3 = 2000$ rpm ($n_3 = 2000$ rpm $< 70\%$ n_1 – critical speed).

Experimental measurement results also show that the rotor operation is stable at these speeds as shown in Figure 7.

TABLE 1. Experimental results of oscillation measurement

No.	Frequency (Hz)	Average speed (rpm)	Displacement	
			X (mm)	Y (mm)
1	19	800	$\pm 0,05$	$\pm 0,12$
2	26	1500	$\pm 0,055$	$\pm 0,11$
3	34	2000	$\pm 0,05$	$\pm 0,10$

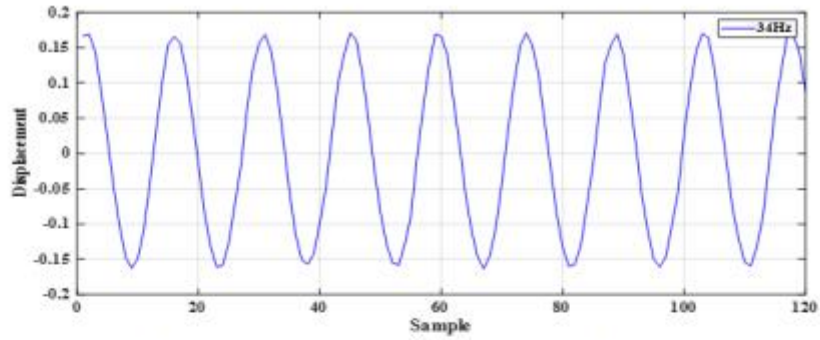
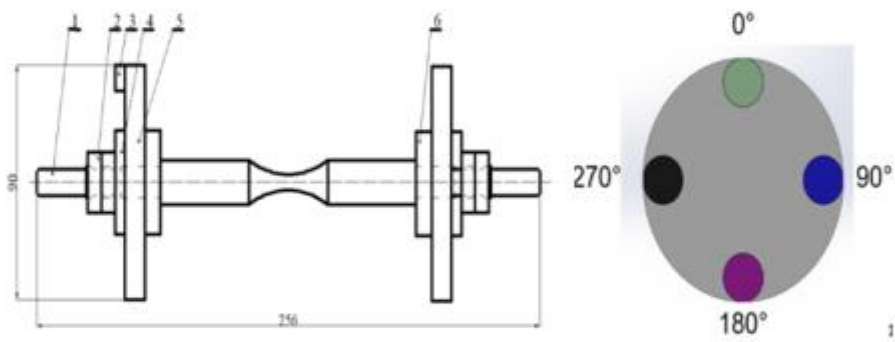


FIGURE 7. Result of displacement measurement at actual speed $n_3 = 2000$ rpm ($f = 34$ Hz)

Effect of Unbalance

Case 1 - Trial mass placed symmetrically position ($0^\circ - 0^\circ$)

The rotor at unbalanced status has a structure as shown in Figure 8.



1. Rotary shaft - 2. Nut - 3. Trial mass - 4, 6. Bushings - 5. Disc - 7. Load position

FIGURE 8. Experimental rotor at unbalanced

According to the unbalanced standard ISO 1940/1 - 2013; we calculate the trial mass corresponding to the unbalanced levels G2.5, G6.3 and G16 [11]

For G16, we have the trial mass for calculation at minimum speed n_1 is:

$$m_{trial} = \frac{k \cdot 9.54 \cdot G_{16} \cdot M_{rotor}}{n_1 \cdot r} = 37.7 \text{ g}$$

Therefore, we choose the trial mass 10g, 20g and 30g respectively and run at the suggested speeds, the measurement results are as shown in Figure 9.

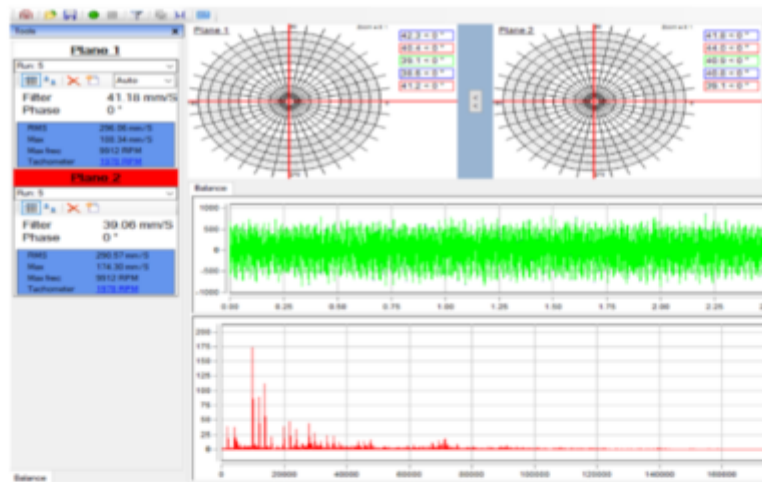


FIGURE 9. Unbalanced measurement results on 2 planes at position $0^\circ - 0^\circ$; $m_{trial} = 30g$.

The results of measuring the amount of unbalance on the rotor at positions $0^\circ - 0^\circ$ with the corresponding load values are presented in table 2.

TABLE 2. Unbalanced measurement results when set trial mass at position $0^\circ - 0^\circ$

No.	Trial mass (gram)	Speed (rpm)	$G=e_p \cdot \omega$ (mm/s)
1	0	800	0.58
2	0	1500	1.12
3	0	2000	1.34
4	10	800	0.9
5	10	1500	4.9
6	10	2000	20.5
7	20	800	1.1
8	20	1500	9.4
9	20	2000	32.8
10	30	800	2.2
11	30	1500	14.3
12	30	2000	42.3

Then the displacement graph is:

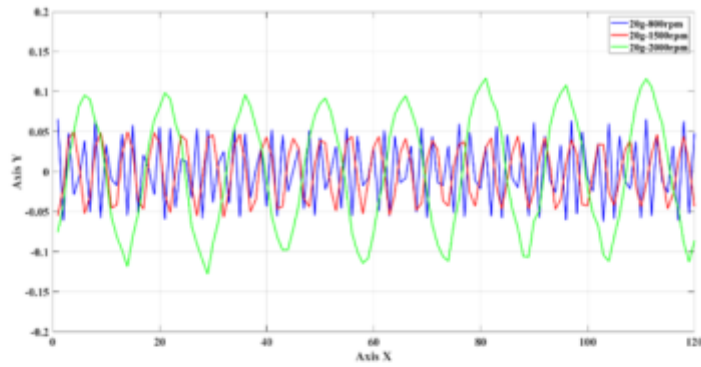


FIGURE 10. Result of displacement measurement at trial mass $m_{trial}=20g$, speed 800, 1500 and 2000 rpm

Comment: at the speed 2000 rpm, the rotor oscillation is the largest ($m = 20g$; $G = 32,8 \text{ mm/s}$)

Case 2 - Asymmetric trial mass (position $0^\circ - 90^\circ$; $0^\circ - 180^\circ$)

Changing the initial load position (phase angle), we get unbalanced results as in table 3.

TABLE 3. Unbalanced measurement results when set the trial mass at position $0^\circ - 90^\circ$, $0^\circ - 180^\circ$

No.	Position of m_{trial}	Trial mass (gram)	Speed (rpm)	$G=e_p \cdot \omega$ (mm/s)
1	$0^\circ - 0^\circ$	10	2000	20.5
2	$0^\circ - 90^\circ$	10	2000	14.3
3	$0^\circ - 180^\circ$	10	2000	2.6
4	$0^\circ - 0^\circ$	20	2000	32.8
5	$0^\circ - 90^\circ$	20	2000	28.7
6	$0^\circ - 180^\circ$	20	2000	3.7
7	$0^\circ - 0^\circ$	30	2000	42.3
8	$0^\circ - 90^\circ$	30	2000	35.4
9	$0^\circ - 180^\circ$	30	2000	4.2

Corresponding, we have a displacement graph as figure 11

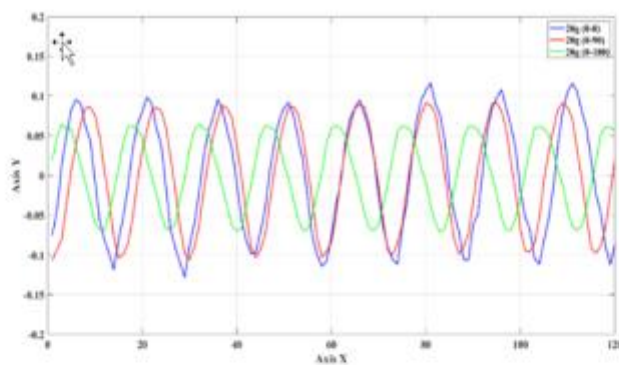


FIGURE 11. Displacement results obtained at trial mass $m_{trial}=20g$, speed 2000 rpm, position $0^\circ - 0^\circ$, $0^\circ - 90^\circ$, $0^\circ - 180^\circ$

The measurement results show that the amount of unbalance and displacement decreases from the position of phase angle $0^\circ - 0^\circ \rightarrow$ position $0^\circ - 90^\circ \rightarrow$ position $0^\circ - 180^\circ$.

Rotor Orbit

The motion orbit of the rotor is shown in figure 12 at speed $n_1 = 2000$ rpm; in the two cases of rotor no-load and rotor have loaded 30g, position $0^\circ - 0^\circ$.

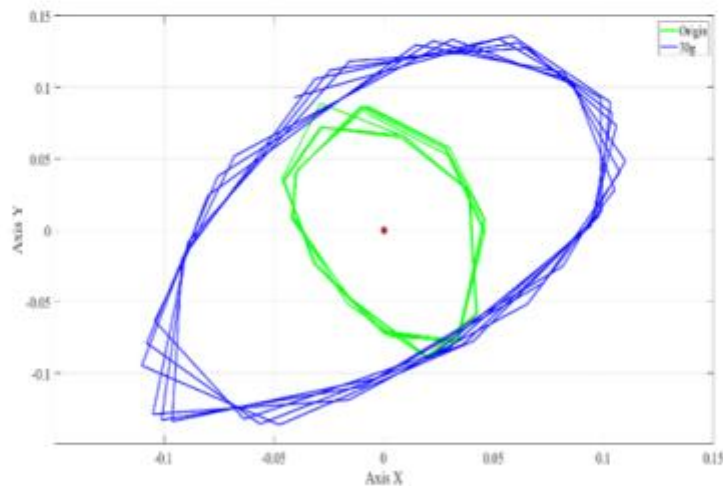


FIGURE 12. The motion orbit of the shaft center corresponds to the no-loaded and loaded rotor 30g.

070003-9

Experimental results show that the motion orbit of the rotor is appropriate behavior with the laws of influence parameters.

CONCLUSION

The rotor vibration evaluation model has been established and operates stably. The influence of many parameters on vibration response, including rotational speed, unbalanced eccentricity, and initial phase angle, is studied.

The results show that:

When the speed around the critical speed, the vibration becomes unstable, the amplitude of oscillation increases sharply, the rotor will be degraded very quickly.

The unbalanced initial phase angle causes different variations of the vibration phase, resulting in placement or superposition of tensile or compressive stresses resulting in fatigue appearing on the shaft.

In actual engineering problems, speed ranges corresponding to the critical speed should be avoided and reduced unbalanced eccentricity, unbalanced direction change, which can slow down rotor damage.

These findings have important implications for guiding the safe design of rotor systems for long-term operation and for further study of prognosis and prediction of rotor life.

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Effect of unbalanced parameters on the fatigue behavior of transmission shaft in gearbox

Strength, Fracture and Complexity
1–7

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Abstract

The rotating shafts in gearboxes operate at various speeds and loads, which can lead to fatigue failure. One of the fundamental causes of damage is vibration due to unbalance, misalignment, or non-uniformity during rotation, which have a detrimental effect on fatigue strength. In this study, the influence of unbalance parameters on the high cycle fatigue behavior of AISI 1045 steel drive shafts was investigated. A completely new fatigue testing equipment is proposed and fully fabricated to examine the factors such as eccentricity due to test mass assignment, operating speed and loading radius affecting fatigue strength of AISI 1045 steel. The results show that the fatigue strength of the unbalanced specimen is lower than that of the pure bending specimen.

Keywords

Fatigue, unbalance, rotating/transmission shaft, dynamic stress, eccentricity

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Introduction

Rotating shafts are essential components of machines, operating under applied loads with periodically bending and torsional stresses. Among several factors contributing to shaft failure, vibration is a primary cause. It can induce fatigue, fractures, and tribological issues within the shaft components. Rubio et al.¹ reported that applied loads combined with frequent mass unbalances, which is one of the most common sources of shaft vibrations, can initiate and propagate fatigue cracks. Mobarak et al.² indicated that a fatigue crack typically generates on a rotating shaft due to applied loads, torsional stresses, and lateral vibrations caused by rotational mass unbalance. They numerically investigated the breathing mechanism of a transverse crack in a two-disc rotor using a three-dimensional finite element model, considering the combined effects of unbalance force and rotor weight. Their findings revealed that crack breathing is highly influenced by various factors, including the axial and angular positions of the crack, the crack depth ratio, the unbalance force ratio, and the angular position of the unbalance force. Mobarak et al.³ developed an unbalanced shaft model to study the effect of crack location on the breathing behavior of fatigue cracks under an unbalanced force. Yongyao et al.⁴ investigated the fatigue of a piston rod in a Kaplan turbine, caused by unsteady,

unbalanced, and unsynchronized blade torques. They concluded that the fractured sections of the piston rod exhibited characteristics of fatigue failure caused by periodically alternating loads. The most likely cause was the unsteady pressure loads acting on the blades. Daging et al.⁵ reported that shaft fatigue occurred due to an unbalance between bending and torsional stresses at the junction of the reinforced-welded elbow.

Niu and Yang⁶ indicated that nonlinear vibration behavior and fatigue crack propagation are significantly influenced by multiple parameters, including rotation speed, unbalance eccentricity, orientation angle, and damping. Radgolchin and Anbarsooz⁷ also reported fatigue is the most common failure mode in centrifugal impellers, caused by high-frequency unbalanced operation, installation errors, or variations in compressor operating

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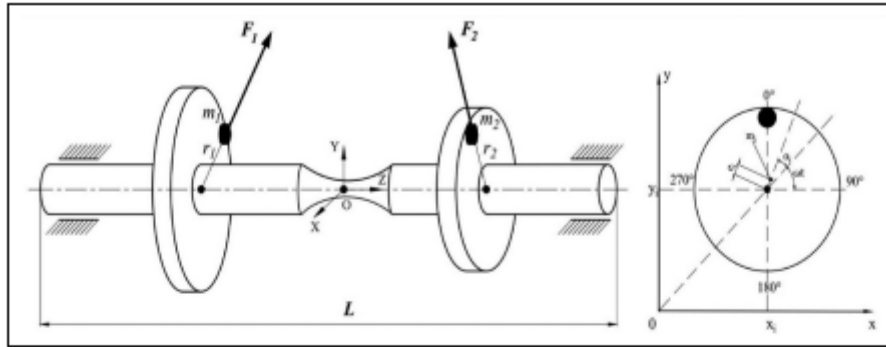


Figure 1. Two – disk model of transmission shaft.

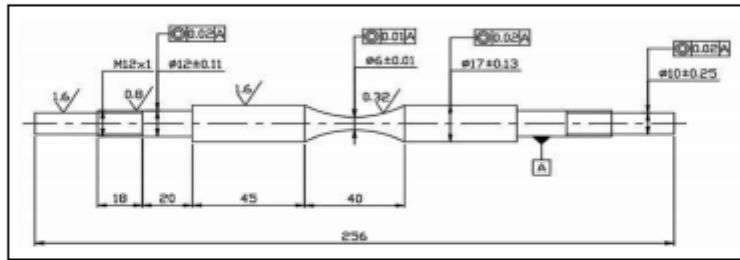


Figure 2. Dimensions of a rotating bending specimen.

conditions. Jiregna et al.⁸ stated that due to rotating bending, fatigue cracks initiate on the surface, particularly when prominent journals become misaligned or unbalanced because of the counterweight effect. Therefore, the unbalanced parameter has a significant impact on fatigue failure.

In this study, the influence of unbalanced factors, including trial mass, operating speed, and load radius on the fatigue behavior of an AISI 1045 steel shaft under rotating bending conditions are investigated. A fatigue curve for a rotating shaft made of AISI 1045 steel, subjected to an eccentric force F_u and a cyclic excitation force F_e , was developed based on experimental data.

Experimental details

The proposed model

To evaluate the influence of parameters causing eccentricity on fatigue strength, a typical model of transmission shaft in gearbox with two disks as shown in Figure 1 are proposed, consisting of multiple shafts and gears, which generate noise and vibration during high-speed operation. This issue stems from the eccentricity e and imbalance of

Table 1. Chemical compositions (wt.%) of AISI 1045 steel.

C	Si	Mn	P	S	Cr	Mo	Ni
0428	0245	0644	0015	0004	0361	0003	0014

rotating components. Prolonged operation under these conditions can result in wear and, eventually, failure due to fatigue damage. F_1 , m_1 , r_1 , F_2 , m_2 , r_2 are the combined forces, trial masses, and load positions of gears 1 and gear 2; ω is the angular velocity.

The combined forces F , F acting on the rotating shaft includes the centrifugal force F and the external excitation force F in a periodic. The centrifugal force F is formed by set the trial mass m at the radius r on disk i: $F_u = m r \omega^2$ and the external excitation force F is controlled by the electromagnet as shown in Figure 3.

Specimens

The material of the specimens adopted in this investigation is an AISI 1045 steel. The chemical composition (wt.%) of AISI 1045 steel was shown in Table 1. In general, the

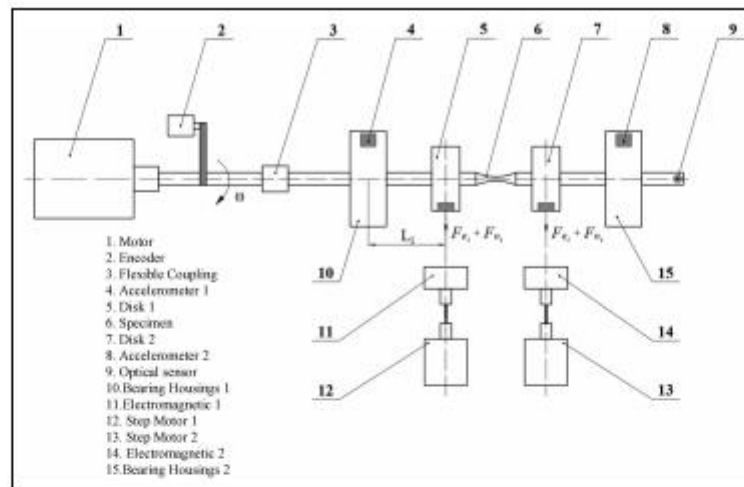


Figure 3. Schematic diagram of fatigue testing machine.

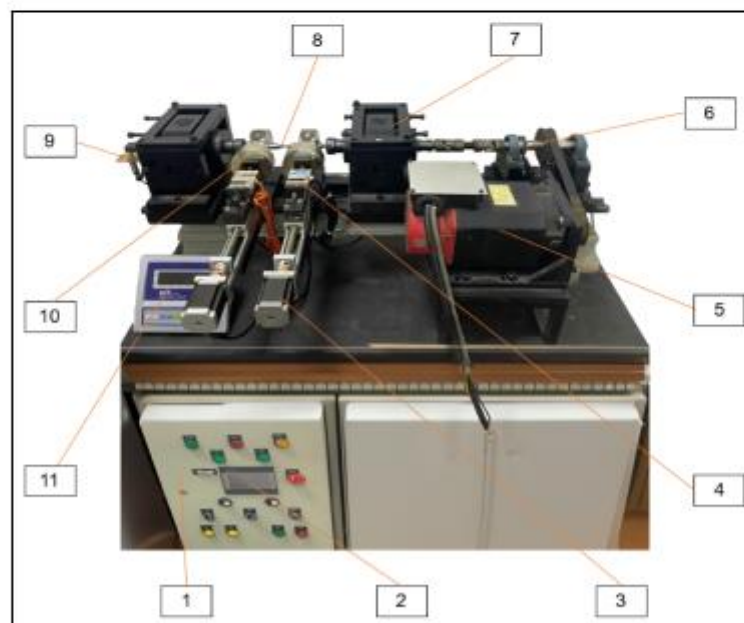


Figure 4. Photograph of fatigue testing machine. 1. Box Control • 2. Human-Machine Interface • 3. Step Motor • 4. Loadcell • 5. Motor • 6. Encoder • 7. Bearing Housing • 8. Specimen • 9. Optical Sensor • 10. Electromagnetic • 11. Display Loadcell.

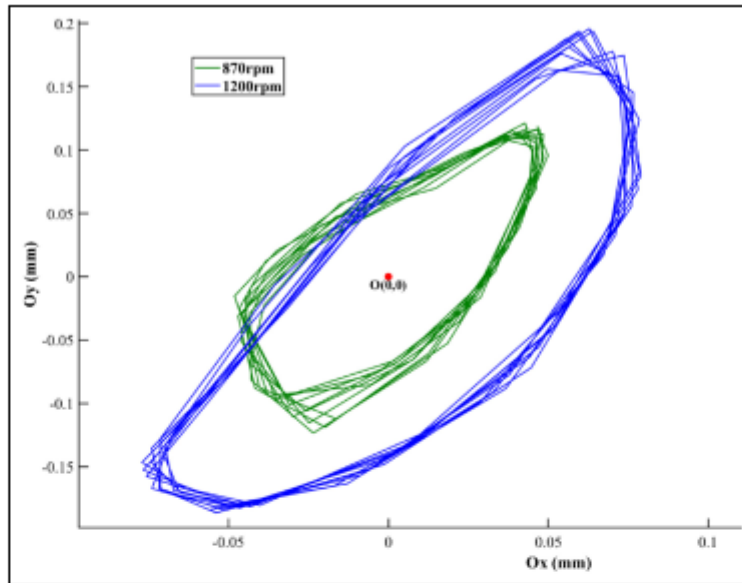


Figure 5. The motional orbit of the center point O at 870 rpm and 1200 rpm.

Table 2. Unbalanced forces due to difference speeds.

No.	m (g)	r (mm)	n (rpm)	F_u (N)	F_s (N)
Point 1	30	50	1200	23.7	10
Point 2	30	50	1100	19.9	10
Point 3	30	50	1010	16.8	10
Point 4	30	50	900	13.3	10
Point 5	30	50	870	12.4	10

hot-rolled steel exhibits anisotropy in its microstructure, texture, and mechanical properties. To eliminate the strong texture and achieve a homogenous and isotropy microstructure, the steel was annealed at 950°C for 15 min. The subsequent step was followed by two additional annealing processes at 840°C and 760°C to improve grain uniformity further.⁹

The mechanical properties of AISI 1045 steel were assessed according to the ISO 6892-1: 2009 tensile testing standard. On average, the tensile strength and yield strength were 706 MPa and 426 MPa, respectively. Figure 2 is the dimensions of specimens according to ISO 1143:2010 standard used for four-point bending fatigue testing.¹⁰ After the final preparation, the specimens were fabricated according to the dimensions shown in Figure 2, achieving a surface roughness R_a of approximately 0.32 μm .

Fatigue test

A novel four-point bending type fatigue testing machine has been newly designed and manufactured to study the fatigue characteristics of unbalanced rotating shafts. Figure 3 shows the schematic diagram of the new fatigue testing machine. Figure 4 is a photograph of the machine. The rotating speed of the spindle can be to 6000 rpm and controlled continuously variable transmission by using an inverter. The applied load onto the discs 1 and 2 are produced and controlled by using electromagnetics 1 and 2, respectively. The applied force (stress) is online measured by loadcell during the experiment. The encoder counts the number of cycles until failure or runout. The fatigue strength is determined when the specimen fractures or 10^7 cycles are completed. Fracture of specimens is defined as the test machine stopping in broken specimens or the number of cycles above a preset level ($>10^7$ cycles).

Results and discussions

Displacement and orbit

Unbalance parameters and unbalanced forces are shown in Table 2. The results indicate that the unbalanced force increases from 12.4 N to 23.7 N as the shaft's rotational speed increases from 870 to 1200 rpm. The electromagnetic force remains constant regardless of speed. Figure 5 shows

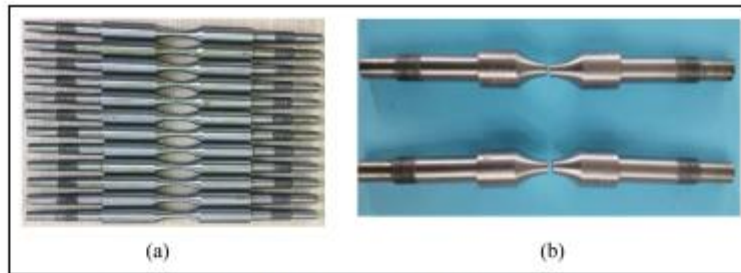


Figure 6. Specimens for fatigue experiments. (a) Original specimen (b) Fractured specimen.

Table 3. Experimental results data of AISI 1045 steel.

	$\sigma_1 = 286$ (MPa)	$\sigma_2 = 254$ (MPa)	$\sigma_3 = 228$ (MPa)	$\sigma_4 = 198$ (MPa)	$\sigma_5 = 190$ (MPa)
Specimen	Cycles ($2N_f \cdot 10^5$)	Cycles ($2N_f \cdot 10^5$)	Cycles ($2N_f \cdot 10^5$)	Cycles ($2N_f \cdot 10^5$)	Cycles ($2N_f \cdot 10^7$)
1	2.41460	6.51057	3.913211	6.885019	Run out
2	2.63141	7.15121	3.151293	5.540163	Run out
3	2.29678	6.11198	3.510518	4.800227	Run out

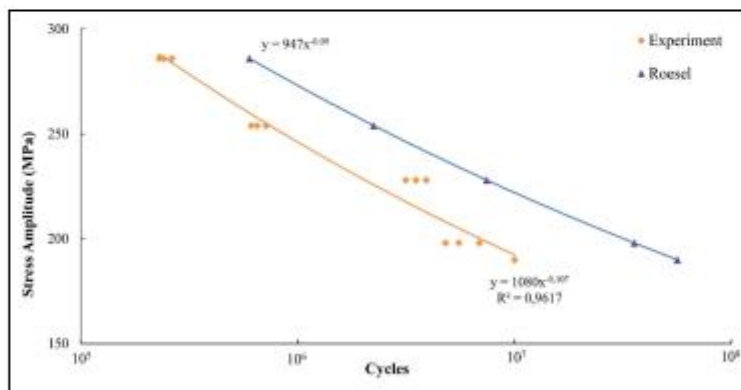


Figure 7. S-N curves for AISI 1045 steel and Roesel's fatigue curves.

the motional orbits of center point O at various speeds n from 870 rpm to 1200 rpm.

The results indicate that increasing rotational speeds cause to larger shaft displacements. The motion orbit of the rotor conforms appropriately to the rules based on the influence parameters. The shaft orbit further indicates that imbalance increases with rising speed. In addition, the orbit motion is larger with the increasing load. This can be explained by the appearance and propagation of

fatigue cracks, which amplify the influence of unbalanced forces on the shaft.

Fatigue experiment

The fatigue tests were carried out on annealed specimens under five values of load, from 190 MPa to 286 MPa. The fractured position is visible in the reduced section of each specimen, indicating that material failure occurred at the center of

Table 4. Basquin law's parameters of AISI 1045 steel under unbalanced forces.

S-N curve	σ'_f	b
Roesle	947	-0.09
Experiment	1080	-0.107

the specimens. Figure 6(a) shows a photograph of a specimen. Figure 6(b) shows the photographs of a fractured specimen. The experimental results are shown in Table 3.

Figure 7 compares the stress amplitude – fatigue life curves (S-N curves) of AISI 1045 steel, determined from experiment and from the pure bending fatigue life of Roesel's model. A simplified Basquin exponential function, such as in equation (1), was used to characterize the fatigue curves.¹¹ The coefficients of equation (1) were determined from the experimental data and reported in Table 4.

$$\sigma_a = \sigma'_f (2N_f)^b \quad (1)$$

The experimental fatigue life for AISI 1045 steel is compared with the pure bending fatigue life of Roesel's model.¹² The results are also shown in Figure 7. The results shows that shaft imbalance greatly affects fatigue resistance. The larger the unbalanced force, the lower the fatigue strength. Therefore, factors causing imbalance need to be controlled in the design and manufacture of machine parts.

Compared to Roesel's model, which was conducted under the same conditions in pure bending, the results indicate that unbalanced parameters affecting the rotating shaft lead to a decrease in fatigue strength compared to the pure bending rotation specimen.

Conclusions

The following conclusions are made:

- A new testing machine for researching fatigue strength of the unbalanced drive shafts has been developed. A novel driver alignment tool was also designed to evaluate these unbalanced effects.
- The fatigue experimental results indicated that the fatigue limit and fatigue strength of the sample decrease under the influence of unbalanced factors; including trial mass, operating speed, and load radius; compared to the pure bending rotating shaft.
- Further research to investigate on the effects of various parameters on the fatigue life of rotating shafts under different operating conditions such as high temperature, high speeds or multi-phase materials should be done.

Nomenclature

F_u	Unbalanced force (N)
F_a	Cyclic excitation force (N)
m	Mass (g)
r	Load radius (mm)
n	Speed (rpm)
ω	Angular velocity (rad/s)
σ_a	Stress amplitude (MPa)
σ'_f	Fatigue strength coefficient (MPa)
$2N_f$	Number of reversals to failure (Cycles)
b	Basquin's exponent

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Statements and declarations

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Conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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Results per page 10

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Investigation on the Influence of Unbalanced Shaft Component in Gearbox on Displacement Using the Newmark- β Method

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Abstract

This study introduces an enhanced numerical approach for analyzing the dynamic behavior of a rotor-bearing system subjected to unbalanced excitation from a gearbox drive shaft. The Newmark- β method with the integration of a variable time-step algorithm was used, allowing the system to be solved rapidly and accurately without compromising stability. This technique enables a precise computation of displacement and torsional deformation of the rotating shaft during its operational cycle. The proposed computational model is validated against experimental data, showing deviations of displacement in normal operation below the critical speed of about 6%. A comprehensive parametric analysis is conducted to evaluate the influence of rotational speed, trial mass, and initial phase angle on the system dynamics. The findings confirm that our enhanced numerical approach yields rapid convergence and reliable predictions, making it a valuable tool for dynamic analysis of rotating systems.

Keywords: Transmission shaft; unbalanced excitation; phase angle; Newmark- β method; dynamic modelling; rotor-bearing system.

1. Introduction

The drive shafts in gearboxes are common components with variety of applications in industries, operating at various speeds and load types. Among many factors causing failure of shafts, vibration due to unbalance is a major cause. Single and coupled vibration modes, including torsional, longitudinal and transverse vibrations induce fatigue, fracture and tribological issues on the rotating shaft components [1-3]. These vibrations result in displacement, performance of gear transmission, wear and cracks [4-6].

Many studies have been carried out on dynamic aspects of the of rotor systems. The modelling of the rotor systems often uses the Jeffcott rotor model having a massless axis and a mass disk placed in the middle of the shaft. Modern analytical methods have been utilized to provide a foundational understanding of rotor dynamics and to conduct simple model experiments [7, 8]. In addition, Yuan et al. [9] and Hong et al. [10] studied a rotor system with a dynamic model, in which a disc is placed in the middle of a massless elastic rotating shaft. The equations of motion are obtained to Lagrangian dynamics for transverse - torsional vibrations. Han and colleagues [11] derived the equation of motion by assuming that the diesel engine drive system can be approached as a simple rotor model such as the Jeffcott rotor. A modified version of this rotor model was also used for analysing the coupled torsional–transverse vibrations of a propeller shaft resulting from misalignment induced by shaft rotation [12]. Besides, Das et al. [13] modelled a flexible rotating shaft system subject to bending and torsion coupled with the shaft and disk moving away from the center point of the shaft. In these studies, it is evident that shaft displacement caused by vibration is a complex problem. Therefore, it is necessary to investigate the shaft's displacement in detail due to unbalance, using a new approach and model in the study of rotor dynamics.

The Newmark- β method, that is a widely used numerical integration technique in finite element analysis, is particularly effective for simulating dynamic systems. A study by Kong et al. used this method to analyze the dynamic characteristics of spur and helical gear systems [14], while a subsequent study focused on the influence of housing flexibility on gear transmission dynamics [15]. Ma and colleagues applied an improved Newmark- β method to determine nonlinear dynamics and reduce crankshaft torsional vibration [16]. An enhanced version of the method was proposed for long-term simulations [17], offering improved convergence in evaluating the effects of nonlinearities on engine crankshaft torsional behavior. Collectively, these studies confirm that the Newmark- β method is a suitable and reliable tool for addressing time-varying oscillation problems.

In general, the current studies lack experimental evidence to support the theoretical computation. This study represents a computational dynamic model for a rotor-bearing system under unbalanced excitation, using the Newmark- β method with a variable time-step algorithm to determine the unbalance amount and shaft displacement. Various key experimental parameters, including rotational speed, trial mass, and initial phase angle, are put into account. The displacement and the amount of unbalance, determined from experiment, are compared with the numerically computational values to verify the validity of the computational model and thus to evaluate their effects on system dynamics. The motional orbit, representing shaft displacement, is also determined to evaluate the influence of the unbalance on the deflection of the shaft. This will allow to predict the fatigue strength, thus to accurately predict performance during operation.

2. Dynamic Model of the Rotor System

2.1. The Proposed Model

This issue arises from the eccentricity and imbalance of rotating components. If the gearbox operates under long-term conditions, it may lead to wear and eventual failure due to fatigue fracture. Fig 1 shows a shaft assembly in the gearbox of a lathing machine, comprising the drive shaft 3, bearings 1 and 5, and gears 2 and 4. Fig 2 represents the schematic diagram of a generalized model comprising a shaft and various assembled disks.

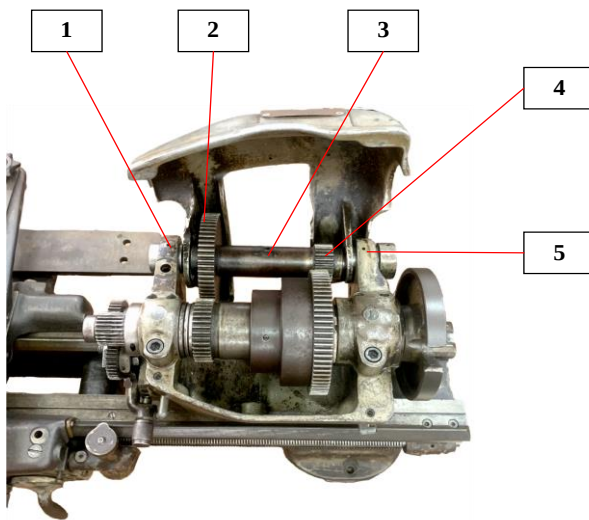
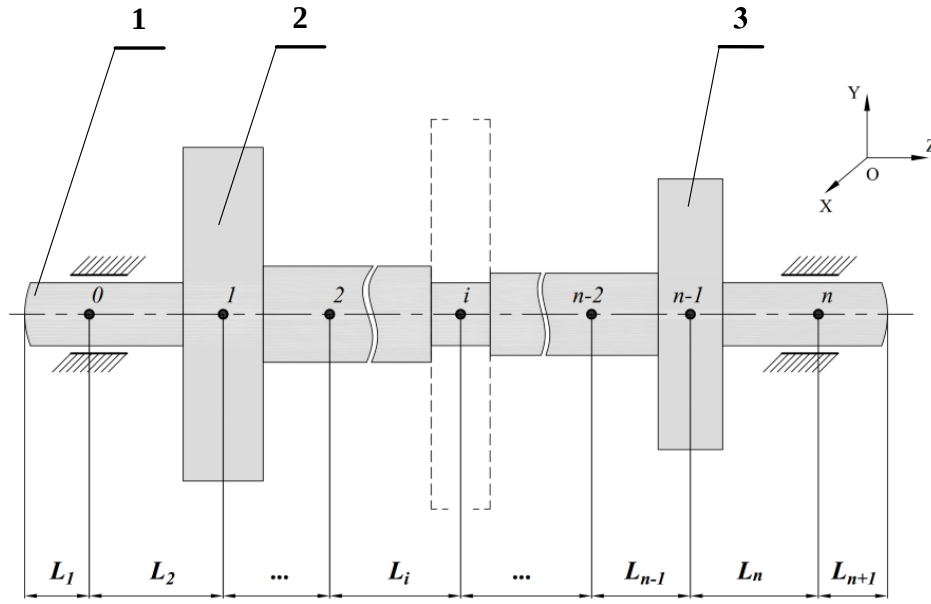


Fig 1. Gear shaft assembly in the lathe gearbox.

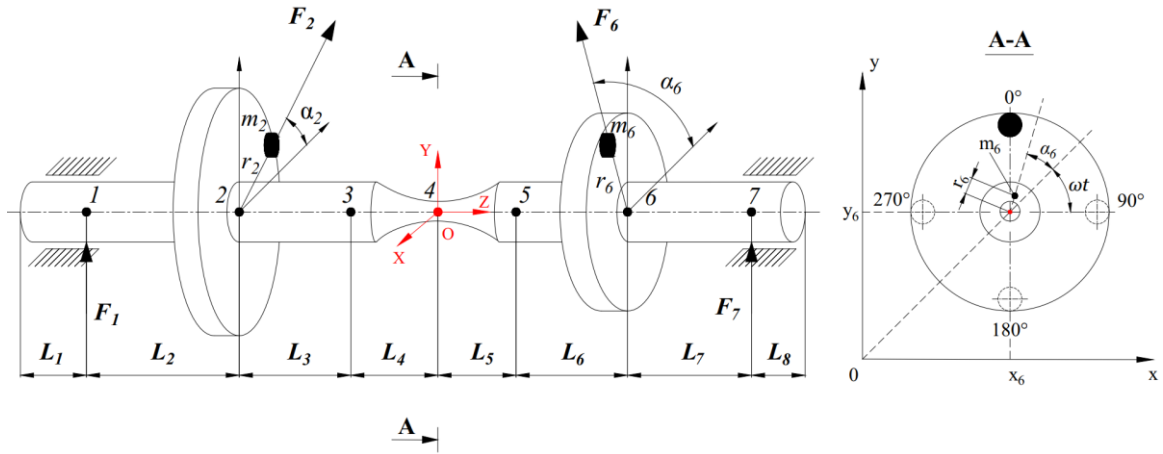


1. Rotating shaft – 2. Disk 1 – 3. Other disks

Fig 2. Schematic diagram of the rotor–bearing system.

2.2. Parameters of the Model

The proposed model of transmission shaft with two disks is shown in Fig. 3. The specimens for evaluating the fatigue strength of metallic materials were used, according to ISO 1143:2010. This specimen type is also well-suited for evaluating location to failure [18]. Seven nodes, from node 1 to node 7, on the rotating shaft are selected to be investigated. Nodes 1 and 7 are the bearing positions. Nodes 2 and 6 are the disk placements and nodes 3 and 5 correspond to the positions with the maximum cross-section on the shaft. Node 4 has the smallest cross-section, where the fracture occurs.



a) Shaft element model with two disks

b) Cross-section showing eccentric mass location

Fig 3. Forces and unbalance positions of rotor-bearing system with eccentric masses

The displacement vector q_i for the shaft nodes i ($i = 1$ to 7) is:

$$q_i = [x_i, y_i, \alpha_i]^T \quad (1)$$

where the displacements x_i and y_i along the X- and Y-axes and torsional angle α_i at the investigated positions are

$$x_i = [x_1, x_2, x_3, x_4, x_5, x_6, x_7]^T; y_i = [y_1, y_2, y_3, y_4, y_5, y_6, y_7]^T$$

$$\alpha_i = [\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7]^T$$

$$\{x_i = x_n \text{ or } x_i = x_m + e_m \cos \varphi_m; y_i = y_n \text{ or } y_i = y_m + e_m \cos \varphi_m :$$

$$n = 1, 3, 5, 7; m = 2, 4, 6$$

Since the loads are applied onto the gears at nodes 2 and 6, the rotation angle on the shaft segment between nodes 2 and 6 are thoroughly investigated. The torsional angles at nodes 3 and 5 are assumed to be small and negligible ($\alpha_3 = \alpha_5 = 0$). Thus, the torsional angles at nodes 2, 4, and 6 are $\alpha_i = [\alpha_2, \alpha_4, \alpha_6]^T$ where $i = 2, 4, 6$. The governing equation for the rotation angle is given as:

$$\phi_i(t) = \alpha_i + \omega_i t + \phi_{0_i} \quad (2)$$

where $\phi_i(t)$ is the total rotation angle at node i as time t ; ϕ_{0_i} is the initial phase angle at node i ; α_i is the torsional angle at node i ; ω_i is the angular velocity at node i ($i = 2, 4, 6$). The kinetic energy of the system is represented as:

$$T = T_t + T_r = \frac{1}{2}\{\dot{x}\}^T[M_c]\{\dot{x}\} + \frac{1}{2}\{\dot{y}\}^T[M_c]\{\dot{y}\} + \frac{1}{2}\{\dot{\phi}\}^T[J_c]\{\dot{\phi}\} \quad (3)$$

where T_t is translational kinetic energy; T_r is rotational kinetic energy; $\{\dot{x}\}$, $\{\dot{y}\}$ are the vectors of translational velocities in the X and Y directions; $\{\dot{\phi}\}$ is the vector of angular velocities of the components; $[M_c]$ is the mass matrix corresponding to the system from node 1 to node 7; $[J_c]$ is the static moments of inertia concerning the study nodes 2, 4, 6. The elastic potential energy of the system is:

$$V_c = \frac{1}{2}\{x\}^T[K_x]\{x\} + \frac{1}{2}\{y\}^T[K_y]\{y\} + \frac{1}{2}\{\alpha\}^T[K_t]\{\alpha\} \quad (4)$$

where $[K_x]$ and $[K_y]$ are respectively the stiffness matrices corresponding to translational stiffness in the X- and Y-axes, $[K_t]$ is the torsional stiffness matrix related to angular displacements. According to Fig. 3, an examination of the shaft segment from node 2 to node 6 represented by l_3 to l_6 was performed. Figure 4 shows the spring modeling of the shaft segment.

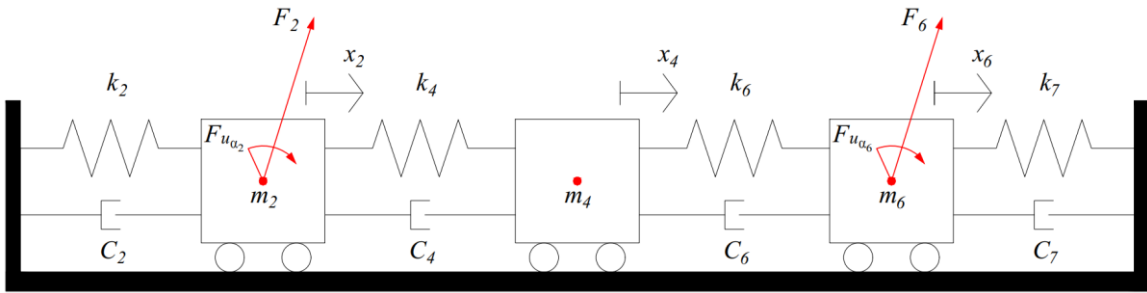


Fig 4. Modelling of the study shaft segment from nodes 2, 4 and 6.

In the torsional shaft problem, x_2 , x_4 , and x_6 are equated to the torsional angles α_2 , α_4 and α_6 of the shaft at nodes 2, 4 and 6, respectively. From the modelling of the shaft segment in Fig. 4, the elastic potential energy is written as:

$$V = \frac{1}{2}k_{t_2}\alpha_2^2 + \frac{1}{2}k_{t_4}(\alpha_2 - \alpha_4)^2 + \frac{1}{2}k_{t_6}(\alpha_4 - \alpha_6)^2 \quad (5)$$

The dissipated energy of the system is given by:

$$D = \frac{1}{2}\{\dot{x}\}^T[C_x]\{\dot{x}\} + \frac{1}{2}\{\dot{y}\}^T[C_y]\{\dot{y}\} + \frac{1}{2}\{\dot{\alpha}\}^T[C_t]\{\dot{\alpha}\} \quad (6)$$

where D is the total energy dissipated due to damping in the system; $[C_x]$, $[C_y]$, and $[C_t]$ are respectively the damping matrices corresponding to translational damping in the X, Y directions and axial torsional damping. The dissipated energy due to damping is written as:

$$D = \frac{1}{2}C_{t_2}\dot{\alpha}_2^2 + \frac{1}{2}C_{t_4}(\dot{\alpha}_2 - \dot{\alpha}_4)^2 + \frac{1}{2}C_{t_6}(\dot{\alpha}_4 - \dot{\alpha}_6)^2 \quad (7)$$

where C_{t_2} , C_{t_4} and C_{t_6} are the torsional damping at nodes 2, 4, and 6. The Lagrange's equations of the second kind can be written in terms of the system as follows [19]:

$$L = T - V \quad (8)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \{\dot{q}_i\}}\right) - \frac{\partial T}{\partial \{q_i\}} + \frac{\partial D}{\partial \{\dot{q}_i\}} + \frac{\partial V}{\partial \{q_i\}} = F_i$$

where T is the total kinetic energy, V is the total potential energy of the system; $F_i = F_{g_i} + F_{u_i}$; F_{g_i} is force due to the mass of the node; F_{u_i} is centrifugal force due to unbalance. The differential equation of motion for the rotor at this time is:

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{F_i\} = \{F_{g_i}\} + \{F_{u_i}\} \quad (9)$$

where $[M]_{17 \times 17}$ is the mass matrix of the system; $[C]_{17 \times 17}$ is damping matrix; $[K]_{17 \times 17}$ is a stiffness matrix. The displacement vector $\{q\}_{17 \times 1}$ includes the displacements in the X-direction, Y-direction, as well as angular rotation, $\{F_i\}_{17 \times 1}$ is excitation force vector:

$$\begin{aligned} \{F_{g_i}\}^T &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ m_1 g \ m_2 g \ m_3 g \ m_4 g \ m_5 g \ m_6 g \ m_7 g \ 0 \ 0 \ 0] \\ \{F_{u_i}\}^T &= [0 \ F_{u_{x2}} \ 0 \ F_{u_{x4}} \ 0 \ F_{u_{x6}} \ 0 \ 0 \ F_{u_{y2}} \ 0 \ F_{u_{y4}} \ 0 \ F_{u_{y6}} \ 0 \ F_{u_{\alpha 2}} \ F_{u_{\alpha 4}} \ F_{u_{\alpha 6}}] \end{aligned} \quad (10)$$

By solving equation (10) for $i = 2, 4, 6$; we obtain:

$$F_{u_{x_i}} = m_i e_i (\dot{\phi}_i^2 \cos \vartheta_i + \ddot{\phi}_i \sin \vartheta_i)$$

$$F_{u_{y_i}} = m_i e_i (\dot{\phi}_i^2 \sin \vartheta_i - \ddot{\phi}_i \cos \vartheta_i)$$

$$F_{u_{\alpha_i}} = m_i e_i (\ddot{x}_i \sin \vartheta_i - \ddot{y}_i \cos \vartheta_i)$$

2.3. Solution Method

Using the Newmark time-stepping analysis method to solve Equation (9), the following variables and matrices are involved: $\{q_i\}$, $\{\dot{q}_i\}$, $[M]$, $[C]$, $[K]$, $\{F_g\}$, $\{F_u\}$, Δt , t_i , γ , β

Take the derivation of Equation (9), we have:

$$\{\ddot{q}_i\} = [M]^{-1}(-[C]\{\dot{q}_i\} - [K]\{q_i\} + \{F_g\} + \{F_u\}) \quad (11)$$

where $\{q_{i+1}\} = \{q_i\} + \Delta t \{\dot{q}_i\} + \left(\frac{1}{2} - \beta\right) \Delta t^2 \{\ddot{q}_i\} + \beta \Delta t^2 \{\ddot{q}_{i+1}\}$ $\{\dot{q}_{i+1}\} = \{\dot{q}_i\} + (1 - \gamma) \Delta t \{\ddot{q}_i\} + \gamma \Delta t \{\ddot{q}_{i+1}\}$

$$M \{\ddot{q}_{i+1}\} + C \{\dot{q}_{i+1}\} + K \{q_{i+1}\} = F_{i+1} \quad (12)$$

Transforming Eq. (12), we obtain:

$$[M + C\gamma\Delta t + K\beta\Delta t^2]\ddot{q}_{i+1} = F_{i+1} - C[\dot{q}_i + (1 - \gamma)\Delta t\ddot{q}_i] - K\left[q_i + \Delta t\dot{q}_i + \left(\frac{1}{2} - \beta\right)\Delta t^2\ddot{q}_i\right] \quad (13)$$

Equation (13) can be generalized as:

$$[\hat{K}]\{\ddot{q}_{i+1}\} = \{\hat{F}_{i+1}\} \quad (14)$$

where $[\hat{K}]$ is the effective stiffness matrix, determined as:

$$[\hat{K}] = [M + C\gamma\Delta t + K\beta\Delta t^2] \quad (15)$$

$\{\hat{F}_{i+1}\}$ is the effective force vector, determined as:

$$\hat{F}_{i+1} = F_{i+1} - C\left[\dot{q}_i + (1 - \gamma)\Delta t\ddot{q}_i\right] - K\left[q_i + \Delta t\dot{q}_i + \left(\frac{1}{2} - \beta\right)\Delta t^2\ddot{q}_i\right] \quad (16)$$

In this study, the implicit Newmark- β method was employed, since it does not require adherence to the time step condition as $\Delta t \leq \frac{2}{\omega_{max}} = \frac{2}{319} = 0.006s$. The parameters γ and β were adjusted during the time-stepping loop and set to $\gamma = \frac{1}{2}$ and $\beta = \frac{1}{4}$. A constant time step of $\Delta t = 0.01s$ was selected for the simulations to ensure stability and computational efficiency [20]. Figure 5 shows the algorithm flowchart, used for computing the displacement q_i , the velocity $\dot{q}_i$, and the acceleration $\ddot{q}_i$.

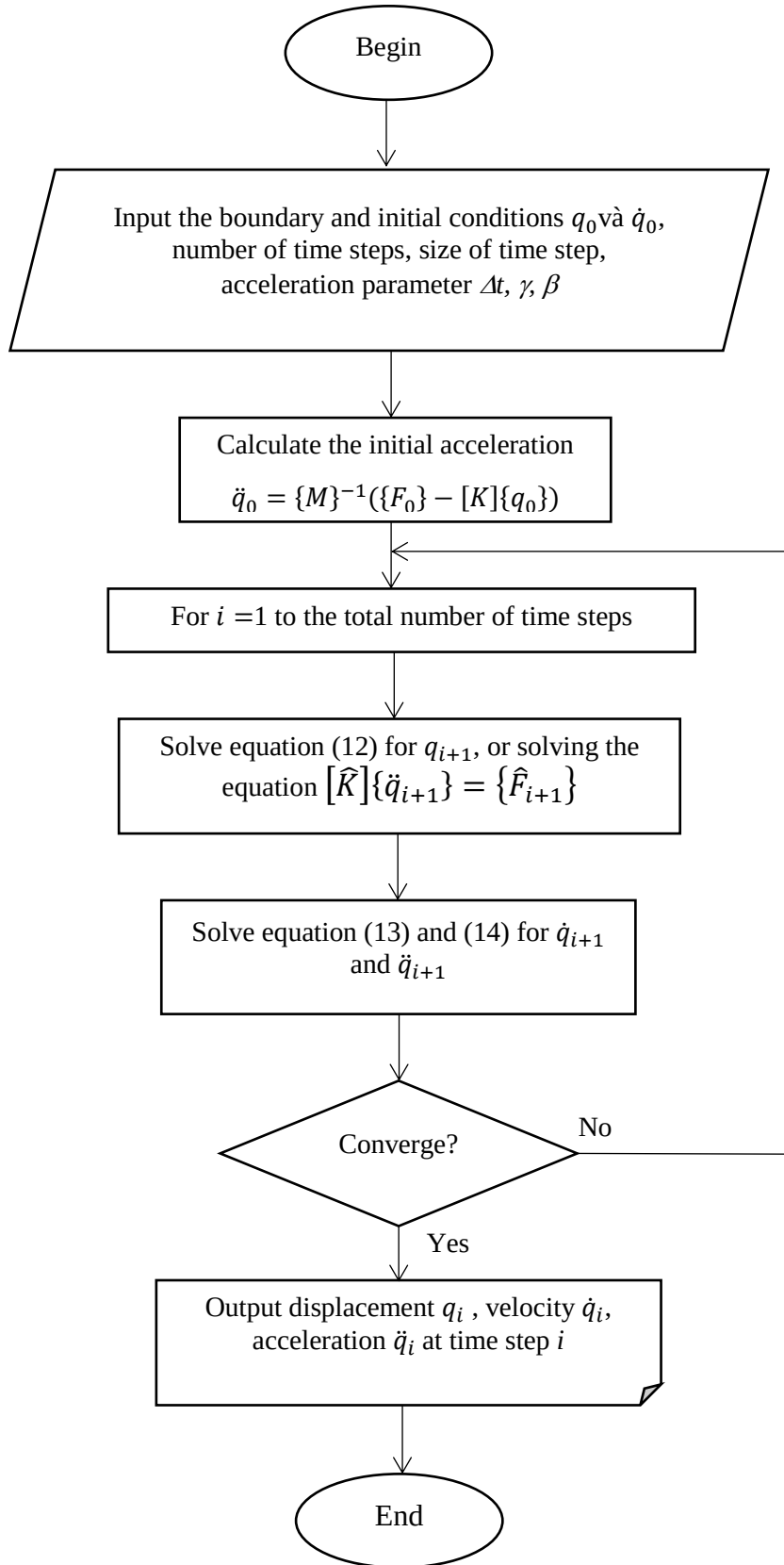


Fig 5. A flowchart for dynamic response of the rotor system.

3. Experimental Procedure

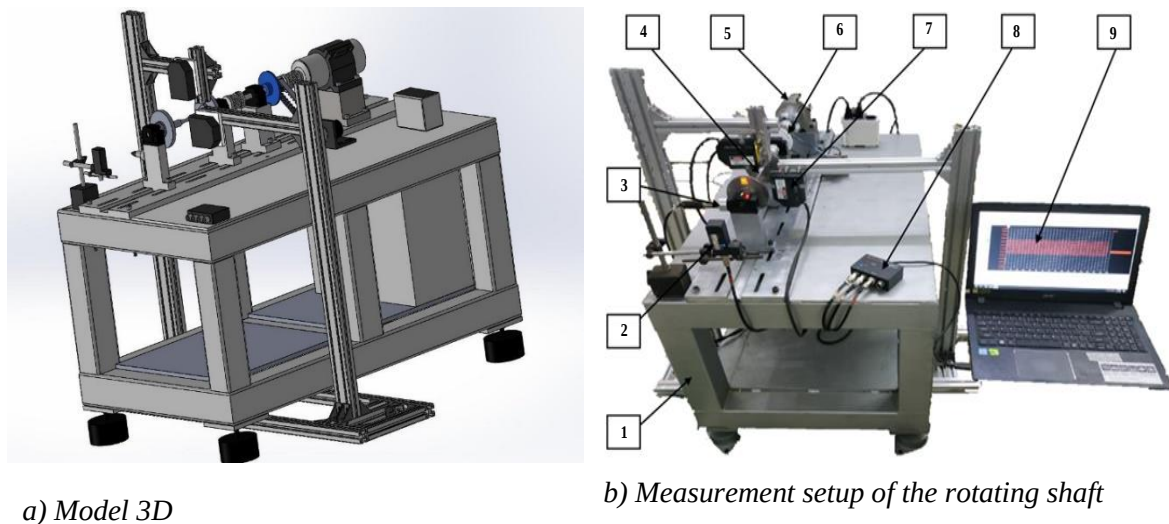
An AISI 1045 steel transmission shaft as shown in Fig 3 with the unbalances m_2 and m_6 was prepared to investigate the behavior of nodes 1 to 7 during rotational operation. Table 1 shows the geometry dimensions and mechanical properties of the shaft [21-23].

Table 1. Symbols, parameters, and unit of power transmission shaft system

No.	Parameters	Symbols	Unit	Value
1	Shaft length	L	m	0.256
2	Section lengths	$l_1 = l_8$	m	0.008
3	Section lengths	$l_2 = l_7$	m	0.045
4	Section lengths	$l_3 = l_6$	m	0.040
5	Section lengths	$l_4 = l_5$	m	0.035
6	Shaft diameter	d_{shaft}	m	0.012
7	Disk diameter	D_{disk}	m	0.1
8	Disk thickness	B	m	0.02
9	Initial phase angle of disk 1	ϕ_{0_2}	rad	0
10	Initial phase angle of disk 2	ϕ_{0_6}	rad	0
11	Unbalance eccentricity at node 2	e_2	m	4.7×10^{-3}
12	Unbalance eccentricity at node 6	e_6	m	4.7×10^{-3}
13	Stiffness of rotor	K_r	N/m	7×10^7
14	Stiffness of bearing	K_b	N/m	7.2×10^6
15	Stiffness of torsional shaft	K_t	Nm/rad	1×10^5
16	Damping of bearing	C	N.m/s	20
17	Friction coefficient	μ		0.1
18	Poisson's ratio	ν		0.3
19	Density	ρ	kg/m ³	7.8×10^3
20	Shaft mass	m_{shaft}	g	285
21	Disk mass	m_{disk}	g	200
22	Rotor mass	m	g	685
23	Masses of nodes 1 and 7	$m_1 = m_7$	g	15
24	Masses of nodes 2 and 6	$m_2 = m_6$	g	35
25	Masses of nodes 3 and 5	$m_3 = m_5$	g	80
26	Mass of node 4	m_4	g	25
27	Gravitational acceleration	g	m/s ²	9.81
28	Moment of inertia	I_p	m ⁴	1.2×10^{-7}

29	Young's modulus	E	N/m ²	2.1×10^{11}
30	Torsional modulus	G	N/m ²	7.7×10^{10}
31	Damping of torsional shaft	C_t	Ns/rad	0
32	Angular velocity	ω_3	rad/s	209

Fig. 6 shows the vibration testing machine, designed and fabricated for study the vibration behavior of the rotating shaft. The shaft was driven by an AC motor and an encoder was attached to the motor spindle to record the actual speed of the rotating shaft. A laser sensor measured the displacement of the rotor. The accelerometer and processor recorded the amount and positions of unbalance on the rotor. The operating tests were performed in controlled modes.



1. Machine Pedestal – 2. Optical Sensor – 3. Acceleration Sensor – 4. Rotating shaft - 5. Motor – 6. Encoder – 7. Measurement sensor LK-G35 – 8. DAQ – 9. Personal Computer

Fig 6. A vibration testing machine of rotating shaft.

4. Results and Discussions

4.1. The Effect of Rotational Speed on Shaft Displacement

In Fig. 7, the rotor oscillates when operated around the first critical speed (mode 1). Under the effect of centrifugal force, the shaft is deflected and the shaft center will oscillate with displacement at position 4 with amplitude y_{max} .

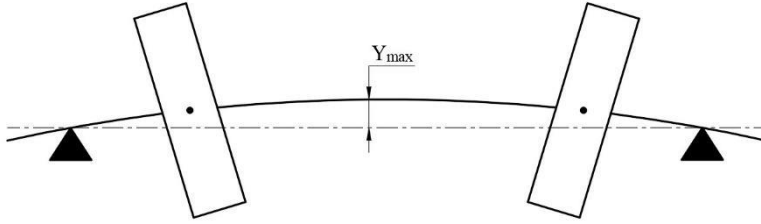


Fig 7. A schematic plot of a rotor operating at the first critical speed (mode 1).

Neglecting the damping coefficient C , the first critical speed of the system can be determined according to [24] using the following expression:

$$\omega_c = \sqrt{\frac{K_r}{m}} = \sqrt{\frac{7 * 10^7}{685}} = 319 \text{ rad/s}$$

In this case, the critical speed is:

$$n_c = \frac{60 * \omega_1}{2 * \pi} = \frac{60 * 319}{2 * 3.14} = 3047 \text{ rpm}$$

To ensure the stability of the rotor in the model and corresponding to the actual working speeds of the equipment, the experimental speed ranges of the rotor were selected as follows: $n_1 = 800$ rpm, $n_2 = 1500$ rpm and $n_3 = 2000$ rpm, in which n_3 is selected to be less than 70% of critical speed of 2132 rpm [25]. Figure 8 shows the displacement of node 4 in the X direction at speed of $n_3 = 2000$ rpm, determined from the experiment and the Newmark- β numerical simulation. Generally, the amplitude measured from the experiment is larger than those using Newmark- β numerical computation because various factors in the manufacturing process may affect the unbalanced conditions.

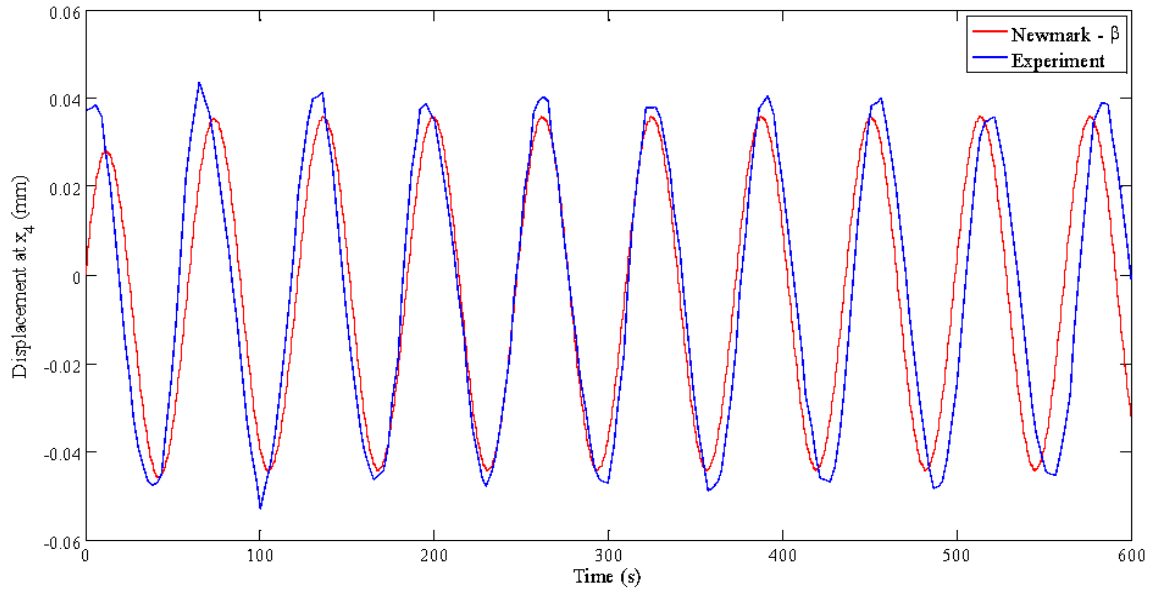


Fig 8. Displacement of node 4 along X-axis at 2000 rpm.

Table 2 compares the X and Y displacement of node 4, determined from the Newmark- β method and from the experiment. The displacement determined experimentally is higher than those determined from the theoretical simulation because the actual stiffness of shaft and bearing is not uniform and due to the inaccuracy and clearance between shaft and bearing. For the operation below the first critical speed, the vibrations remain stable, with displacement amplitudes of $X = \pm 0.046$ and $Y = \pm 0.081$. The relative error of displacement in the X- and Y-axes for speeds of 2000 rpm is 6% and 12%, respectively. As the rotor reaches the first critical speed of 3000 rpm, the vibrational amplitude significantly increases, with displacements of $X = \pm 0.101$ and $Y = \pm 0.119$ mm because of the resonance effects of the rotor components. In general, the displacement in the direction Y is higher than in the X direction because of the gravity force of the components, acting downward, resulting in the higher vertical vibration. The relative errors in displacements are higher by 12% and 18%.

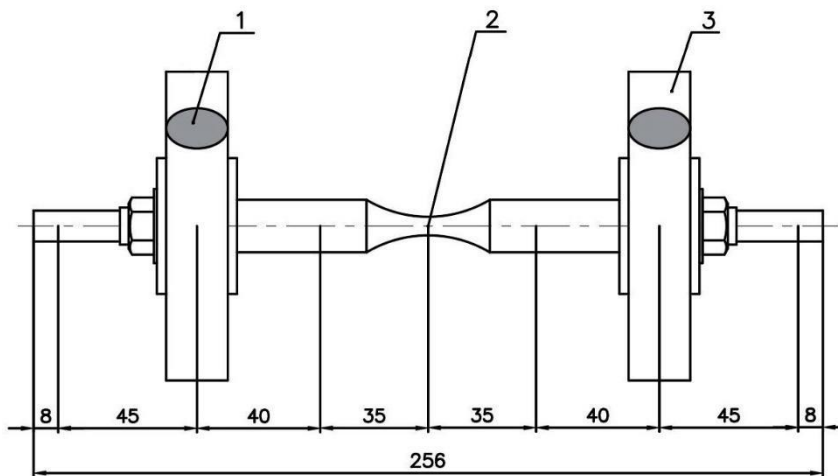
Table 2. Displacement in the X and Y axes at node 4

Frequency (Hz)	Speed (rpm)	Newmark- β method		Experimental data		Relative Error	
		X (mm)	Y (mm)	X (mm)	Y (mm)	X	Y
34	2000	± 0.043	± 0.071	± 0.046	± 0.081	6 %	12 %
51	3000	± 0.085	± 0.097	± 0.101	± 0.119	15 %	18 %

A comparison between the numerical results obtained using the Newmark- β method and the experimental data indicated that the displacement deviation of approximately 6%, which is acceptable for engineering analysis and validates the accuracy of the computational model.

4.2. The Effect of Unbalance on Shaft Displacement

In engineering, for some design circumstance or unavoidable eccentricity in the manufacturing process, the unbalance exists and should be taken into account. The unbalance, eccentricity, and angular position (the angle between the planes of trial masses m_2 and m_6) will be analyzed hereafter. The masses m_2 and m_6 were mounted at three positions 0° – 180° , 0° – 90° , and 0° – 0° such that the direction of m_6 respectively makes angular positions of 180° , 90° and 0° to the direction of m_2 , as shown in Fig 9.



1. Trial mass – 2. Rotating shaft – 3. Disks

Fig 9. Dimension of experimental unbalanced specimen.

4.2.1. Symmetrical Trial Mass (Angular Position: 0°–0° on both disks)

The structure of the unbalanced rotor is shown in Fig. 9, where the trial masses m_2 and m_6 are in the same direction. According to the unbalance standard, the corresponding test masses considered in the range follows as $G1$, $G2.5$, $G6.3$ and $G16$ [26]. For $G1$, the allowable eccentricity e ($n_3 = 2000$ rpm) is:

$$e = \frac{G}{\omega} = \frac{1}{209} = 4.78 * 10^{-3} m$$

At the minimum speed n_1 , the allowable test mass for $G16$ is calculated as follows:

$$m_{trial} = \frac{k * 9.54 * G_{16} * M_{rotor}}{n_1 * r} = 26.1 g$$

Three kinds of trial masses 10 g, 20 g, and 30 g and rotational speeds of 800, 1500 and 2000 rpm were selected to determine the unbalanced, using the commercial Erbesd - Instruments interface, as shown in Fig 10. For the highest trial mass m_{trial} of 30 grams, the unbalanced amounts are respectively 41.18 mm/s and 39.06 mm/s for two nodes 2 and 6.

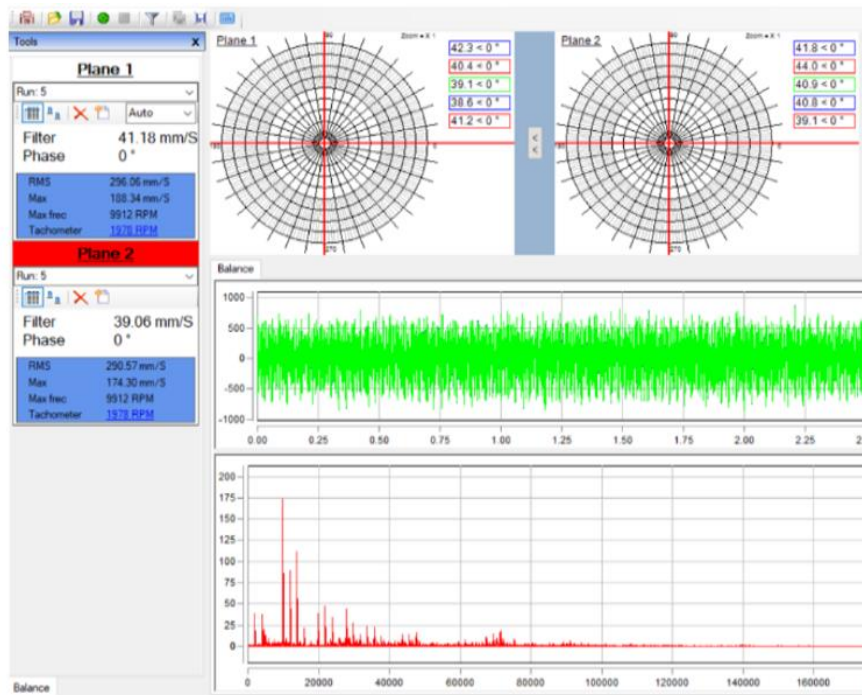


Fig 10. The measurement results of balancing used to two planes at the angular position 0°-0°; $m_{trial} = 30$ grams.

Table 3 represents the measurement results of rotor unbalance at angular position 0^0-0^0 with corresponding to load values and speeds. The corresponding unbalance significantly increases with the increase of trial mass m_{trial} and the rotational speed n respectively. The unbalanced amount G was highest of approximately 41.1 mm/s at n of 2000 rpm with m_{trial} of 30 g and got lowest of 0.58 mm/s at 800 rpm and m_{trial} of 0 g.

Table 3. Unbalance measurement results when attaching trial mass at angular position 0^0-0^0 .

Trial mass m_{trial} (grams)	Speed (rpm)	Amount of unbalance G (mm/s)
0	800	0.58
	1500	1.12
	2000	1.34
10	800	0.9
	1500	4.9
	2000	20.5
20	800	1.1
	1500	9.4
	2000	32.8
30	800	2.2
	1500	14.3
	2000	41.1

Fig 11 and Table 4 represent the variation in horizontal X-axis displacement at node 4 under different operating speeds. The results clearly show the displacement amplitude increase with the rotational speed, particularly as the rotor approaches its first critical speed. This behavior reflects a corresponding rise in dynamic excitation, which is characteristic of resonance phenomena in rotor-dynamic systems.

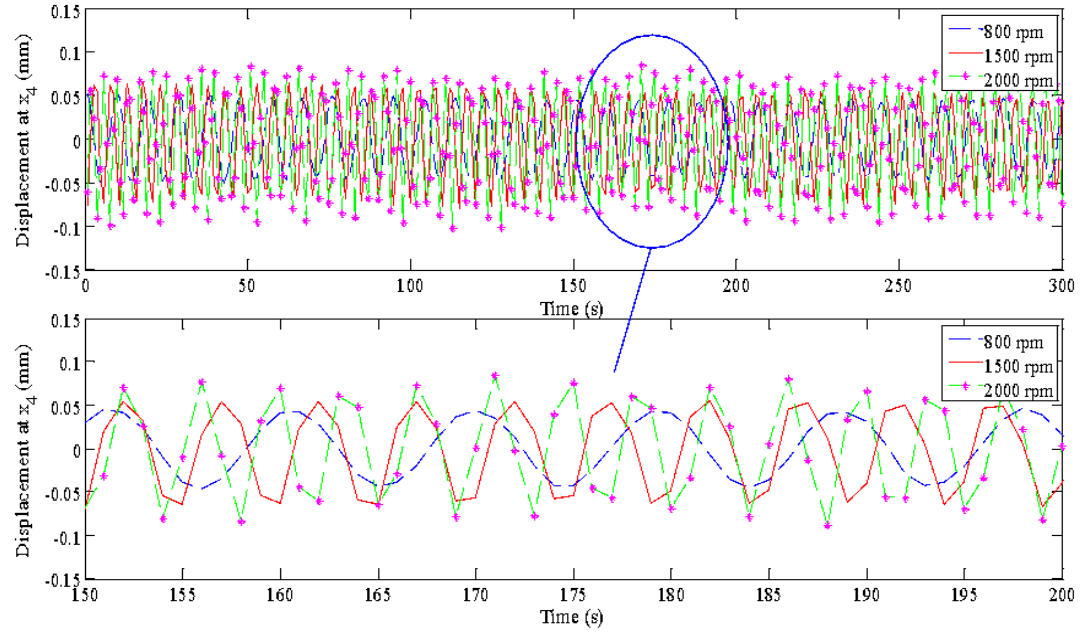


Fig 11. Horizontal displacement of node 4 for $m_{trial} = 20$ g at various speeds.

Table 4. Displacement measurement results at node 4 for 20g trial mass.

m_{trial}	Speed	Displacement at x_4
(grams)	(rpm)	(mm)
20	800	± 0.055
	1500	± 0.073
	2000	± 0.098

4.2.2. Asymmetrically Placed Trial Mass (angular positions 0^0 - 90^0 and 0^0 - 180^0)

By changing the initial phase angle, the results of rotor unbalance at positions 0^0 - 90^0 and 0^0 - 180^0 are shown in Table 5. At a constant rotational speed of 2000 rpm, variations in the load angular position 0^0 - 0^0 , 0^0 - 90^0 , and 0^0 - 180^0 and trial mass m_{trial} of 10, 20, and 30g result in corresponding changes in the component unbalance. The experimental results indicate that the lowest unbalance of 2.6 mm/s occurs at a trial mass of 10 grams and angular positioned at 0^0 - 180^0 . This is because the centrifugal forces of m_2 and m_6 canceled out. On the contrary, the unbalance G obtained the highest value of 42.3 mm/s at the mass 30g and angular positioned at 0^0 - 0^0 because of the resonance of centrifugal forces of m_2 and m_6 .

Table 5. Unbalance G and displacement at 2000 rpm for various trial masses and angular positions.

Angular Position	Trial mass m_{trial} (grams)	Amount of unbalance G (mm/s)	Displacement at x_4 (mm)	Displacement at y_4 (mm)
Original (without trial mass)	0	1.34	± 0.046	± 0.081
$0^0 - 0^0$	10	20.5	± 0.073	± 0.100
$0^0 - 90^0$		14.3	± 0.064	± 0.098
$0^0 - 180^0$		2.6	± 0.050	± 0.085
$0^0 - 0^0$	20	32.8	± 0.098	± 0.111
$0^0 - 90^0$		28.7	± 0.079	± 0.105
$0^0 - 180^0$		3.7	± 0.054	± 0.090
$0^0 - 0^0$	30	42.3	± 0.109	± 0.135
$0^0 - 90^0$		35.4	± 0.090	± 0.111
$0^0 - 180^0$		4.2	± 0.054	± 0.096

In addition, Fig. 12 and Table 6 show a decreasing trend of displacement amplitudes by ± 0.098 mm, ± 0.079 mm, and ± 0.054 mm as the load position shifts from 0^0-0^0 to 0^0-90^0 and then to 0^0-180^0 , respectively.

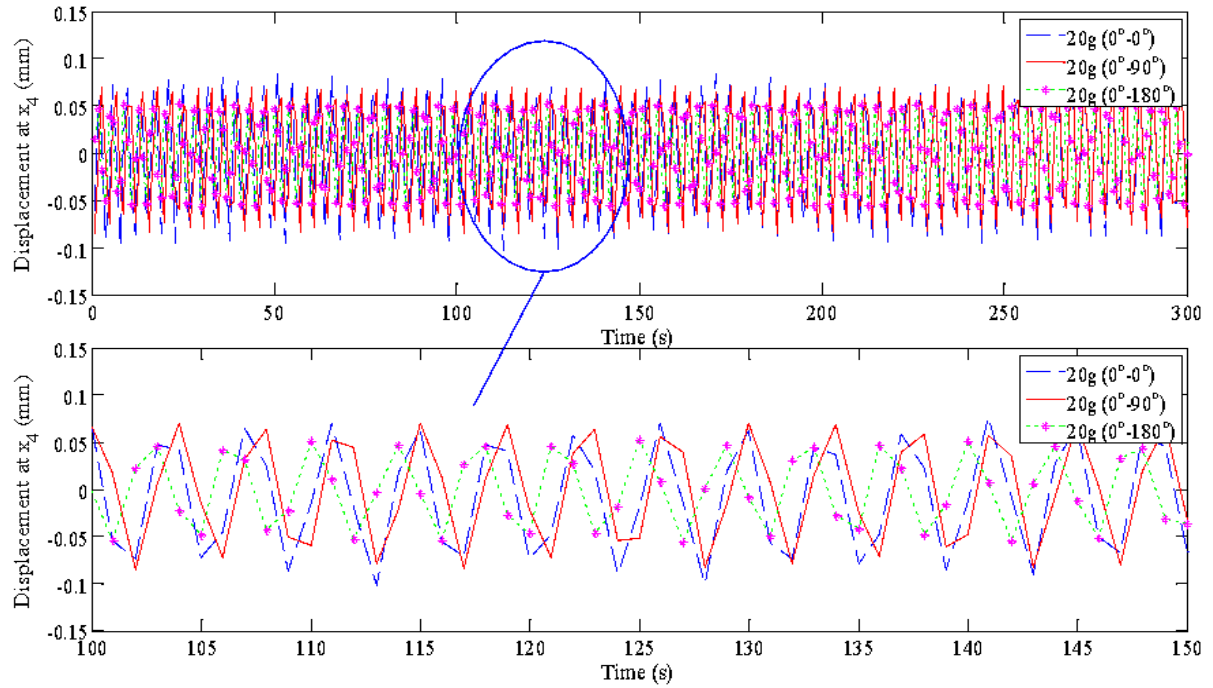


Fig 12. Horizontal displacement of node 4 under 20 g trial mass and speed of 2000 rpm, for the different angular positions: 0^0-0^0 , 0^0-90^0 , and 0^0-180^0 .

Table 6. Displacement measurement results under 20g load at 2000 rpm, with different load angular positions: 0^0-0^0 , 0^0-90^0 , and 0^0-180^0 .

Speed (rpm)	Angular Position	Displacement at x_4 (mm)
2000	$0^0 - 0^0$	± 0.098
	$0^0 - 90^0$	± 0.079
	$0^0 - 180^0$	± 0.054

4.3. Motional Orbit

To evaluate the influence of the unbalanced mass position on the rotor's motion trajectory, a series of experiments was conducted at a rotational speed of 2000 rpm. A trial mass 30g was

mounted on the two discs of the rotating shaft, with various combinations of phase angular positions between the loads. Three unbalanced mass distribution cases were investigated for three angular positions $0^\circ - 0^\circ$, $0^\circ - 90^\circ$ and $0^\circ - 180^\circ$. The motion orbit of the shaft center at the speed $n_3 = 2000$ rpm is represented in Fig. 13. The orbits for load-free rotation and the unbalanced masses in the opposite angular directions $0^\circ - 180^\circ$ obtains the smallest value, while the orbit for angular position $0^\circ - 0^\circ$ is most unstable. This is because of the cancelation or resonance of centrifugal forces of the masses, well agreeing with the experimental displacements in Table 5. Furthermore, the data indicate that the oscillation observed with a trial mass of 30g for the angular position $0^\circ - 0^\circ$ closely resembles the shaft behavior at its first critical speed.

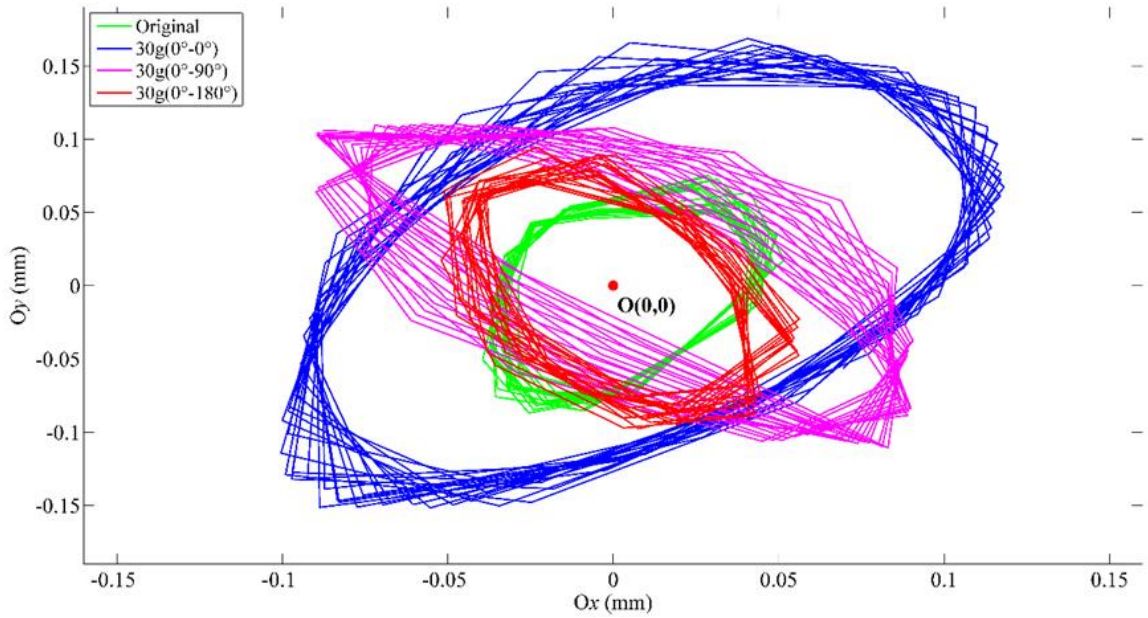


Fig 13. Motional orbit of the shaft center at 2000 rpm for various angular positions

It can be observed that changes in the angular positions of the trial mass $0^\circ - 0^\circ$, $0^\circ - 90^\circ$, and $0^\circ - 180^\circ$ strongly affect the vibration amplitude, thus the shape of the shaft center orbit and well agrees to the expected behavior predicted based on the influence parameters. The greater amount of unbalance and their positions concentration will generate stress accumulation and micro-cracks, resulting in shaft failure.

5. Conclusions

The following conclusions are made:

- a. The effects of such key parameters as rotation speed, eccentricity due to unbalanced and initial phase angle during operation were simulated. The experimental equipment was built and experimental results were compared with the simulated data.
- b. The vibration becomes severely unstable when rotation speed approaches the critical speed, as shown in Tables 3 and 4. The displacement amplitude significantly increases with the amount of unbalance, as shown in Table 5.
- c. The initial phase angle of the trial mass position has great influence on the vibration of the rotating shaft, thus the fatigue bending strength and performance of the component, as shown in Tables 5 and 6 and Fig. 13. By determining the vibration and unbalance conditions of the shaft component, the fatigue limitation can be predicted to ensure the safety and long-term stability of the rotor system.
- d. For operation under the critical speed, the unbalance analysis using the Newmark- β method well agrees with the experimental result, as represented in Table 2, showing that the Newmark- β method is a reliable and stable approach for analyzing the behavior of rotor-bearing systems. This allows rapid and high computational efficiency to predict the shaft operation with fast convergence time.

Further research may be proceeded on the following issues:

- i. Unbalance analysis for double-phase and composite materials.
- ii. Analysis for materials with thin films, coating layer or the surface layer with residual stress.
- iii. Unbalance analysis in high temperature conditions.

Supporting information: Experimental data for Figures 8, 11, 12, and 13.

Author Contributions: Conceptualization, Thanh Lam Tran; Programing, Thanh Lam Tran & Vinh Phoi Nguyen; Specimen manufacturing, Thanh Lam Tran; Data Curation, Thanh Lam Tran; Writing—Original Draft Preparation, Vinh Phoi Nguyen & Thanh Lam Tran; Supervision, Writing—Review & Editing, Chi Cuong Le & Thien Ngon Dang.

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Competing Interests: The authors declare that there is no conflict of academic interest in the study.

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